

## Lab #14: Two-Way ANOVA Key

### 1) 1. Research Questions

1. Does exercise type affect sit-up performance?
2. Is gender related to sit-up performance?
3. Does the effect of exercise type depend on gender?

### 2. Hypotheses

#### Factor A:

	Symbols	Words
H <sub>o</sub>	$\alpha_1 = \alpha_2 = \alpha_3$	Exercise type has no effect on sit-ups.
H <sub>a</sub>	Not H <sub>o</sub>	Exercise type has an effect on sit-ups.

#### Factor B:

	Symbols	Words
H <sub>o</sub>	$\beta_1 = \beta_2$	Gender is unrelated to sit-ups.
H <sub>a</sub>	Not H <sub>o</sub>	Gender is related to sit-ups.

#### A x B Interaction:

	Symbols	Words
H <sub>o</sub>	$\alpha\beta_{11} = \alpha\beta_{12} = \alpha\beta_{13}$ $= \alpha\beta_{21} = \alpha\beta_{22} = \alpha\beta_{23}$	Effects of exercise type does not depend on gender.
H <sub>a</sub>	Not H <sub>o</sub>	Effects of exercise type depends on gender.

### 3. Assumptions

1. H<sub>o</sub>
2. Groups are sampled randomly.
3. Population distribution of the DV is normal in shape.
4. Groups are independent.
5. Population variances are homogenous.
6. N's are equal and greater than 1.
7. Factors are fixed.

### 4. Decision rules

We have a 3x2 factorial design with 5 subjects per group. Thus:

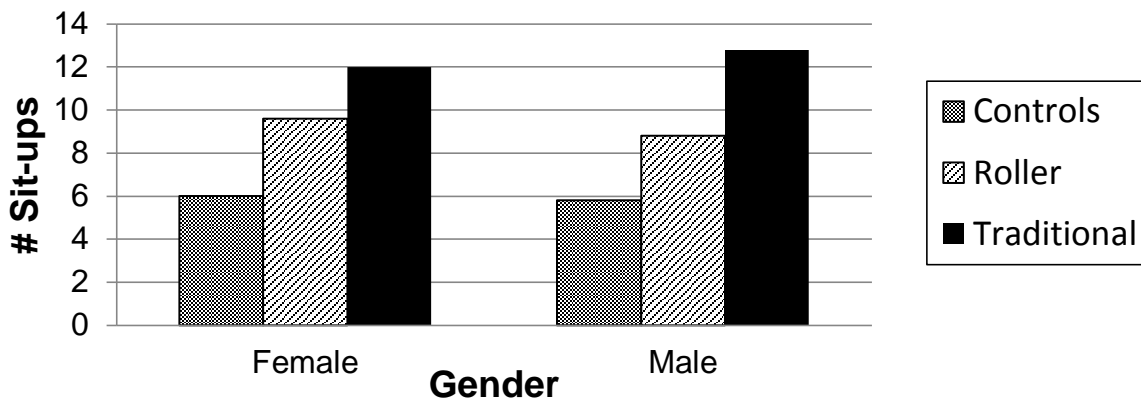
df <sub>A</sub>	=p-1	=3-1	=2
df <sub>B</sub>	=q-1	=2-1	=1
df <sub>AxB</sub>	=(p-1)(q-1)	=(3-1)(2-1)	=2
df <sub>Within</sub>	=pq(n-1)	=(3*2)(5-1)	=24
df <sub>Total</sub>	=npq-1	=(5*3*2)-1	=29

And:

Source	A	B	AxB
df	2/24	1/24	2/24
F <sub>crit</sub>	3.40	4.26	3.40

5. Computation

Gender	b <sub>1</sub>						b <sub>2</sub>						
	a <sub>1</sub>		a <sub>2</sub>		a <sub>3</sub>		a <sub>1</sub>		a <sub>2</sub>		a <sub>3</sub>		
Exercise	X	X <sup>2</sup>	X	X <sup>2</sup>	X	X <sup>2</sup>	X	X <sup>2</sup>	X	X <sup>2</sup>	X	X <sup>2</sup>	
Data	5	25	7	49	9	81	5	25	9	81	11	121	
	7	49	10	100	13	169	8	64	4	16	14	196	
	8	64	11	121	12	144	5	25	12	144	9	81	
	5	25	12	144	11	121	9	81	9	81	16	256	
	5	25	8	64	15	225	2	4	10	100	14	196	
$\sum T_{jk}$	30		48		60		29		44		64		275=T
n <sub>jk</sub>	5		5		5		5		5		5		30=N
$\bar{x}_{jk}$	6.0		9.6		12.0		5.8		8.8		12.8		
$\sum_{i=1}^n X_{ijk}^2$	188		478		740		199		422		850		2877=II



Compute the marginal sum table:

	a <sub>1</sub>	a <sub>2</sub>	a <sub>3</sub>		
b <sub>1</sub>	30	48	60	138	T <sub>.k</sub> 's
b <sub>2</sub>	29	44	64	137	
	59	92	124	275	T <sub>..</sub>
	T <sub>j</sub> 's				

And:

$$\begin{aligned}
 \text{I. } & \frac{T_{..}^2}{N} = \frac{275^2}{30} = \frac{75625}{30} = 2520.833 \\
 \text{II. } & \sum_{i=1}^n \sum_{j=1}^p \sum_{k=1}^q (X_{ijk})^2 = 2877 \\
 \text{III. } & \frac{\sum_{j=1}^p T_{.j}^2}{nq} = \frac{(59^2 + 92^2 + 124^2)}{(5 \cdot 2)} = \frac{(3481 + 8464 + 15376)}{10} = \frac{27321}{10} = 2732.1 \\
 \text{IV. } & \frac{\sum_{k=1}^q T_{.k}^2}{np} = \frac{(138^2 + 137^2)}{(5 \cdot 3)} = \frac{(19044 + 18769)}{15} = \frac{37813}{15} = 2520.867 \\
 \text{V. } & \frac{\sum_{j=1}^p \sum_{k=1}^q T_{jk}^2}{n} = \frac{(30^2 + 48^2 + 60^2 + 29^2 + 44^2 + 64^2)}{5} \\
 & = \frac{(900 + 2304 + 3600 + 841 + 1936 + 4096)}{5} = \frac{13677}{5} = 2735.4
 \end{aligned}$$

Thus:

<b>SS<sub>A</sub></b>	= III-I	= 2732.1 - 2520.833	= 211.267
<b>SS<sub>B</sub></b>	= IV-I	= 2520.867 - 2520.833	= .033
<b>SS<sub>AxB</sub></b>	= V+I-III-IV	= 2735.4 + 2520.833 - 2732.1 - 2520.867	= 3.267
<b>SS<sub>W</sub></b>	= II-V	= 2877 - 2735.4	= 141.6
<b>SS<sub>T</sub></b>	= II-I	= 2877 - 2520.833	= 356.167

$$\mathbf{MS} = \frac{SS}{df} \text{ and } \mathbf{F} = \frac{MS_{\text{source}}}{MS_{\text{error}}}$$

$$\mathbf{MS}_W = \frac{141.6}{24} = 5.9$$

$$\mathbf{MS}_A = \frac{211.267}{2} = 105.633 \text{ and } \mathbf{F}_A = \frac{105.633}{5.9} = 17.9$$

$$\mathbf{MS}_B = \frac{.033}{1} = .033 \text{ and } \mathbf{F}_B = \frac{.033}{5.9} = 0.01$$

$$\mathbf{MS}_{AxB} = \frac{3.267}{2} = 1.633 \text{ and } \mathbf{F}_{AxB} = \frac{1.633}{5.9} = 0.28$$

Source	SS	df	MS	F	P
<b>A</b>	211.267	2	105.633	17.9	< .05
<b>B</b>	.033	1	.033	.01	> .05

<b>AxB</b>	3.267	2	1.633	.28	> .05
<b>Within</b>	141.6	24	5.9		
<b>Total</b>	356.167	29			

**6. Decisions**

1. The main effect of exercise type says that sit-up ability is increased the most with traditional sit-ups, less with the Roller machine, and the least in the control group.
2. The lack of a main effect of gender means that males and females were equivalent in sit-up performance.
3. The lack of an interaction means that the effect of exercise type does not depend on gender.

2) a. 9                      b. 5                      c. 1                      d. 24

3) a

4) e