Lab #13: One-Way ANOVA Key

1) a. Maryanne is misreading the data. She should be comparing the observed F ratio to the F critical value to determine if it is significant. Since Minitab gives the exact probability of F_{obs}, we do not even need F_{crit}. In this case, the p value is .015 which is less than the alpha level of .05

b. The F ratio determines if there is a treatment effect between/among the samples.

$$F = \frac{MS_{Between}}{MS_{Within}} = \frac{\text{Treatment Effect \& Error}}{\text{Error}}$$

If the F ratio is about 1, it suggests no treatment effect.

c. The conclusion from the ANOVA is that the drug matters. Given that the omnibus or overall F ratio is significant we need to do post hoc analyses to localize the effect. The formula for such a comparison is given by:

$$\mathbf{F}_{Comp} = \frac{(\bar{X}_1 - \bar{X}_2)^2}{MS_w\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}$$

It is noteworthy, that the F_{crit} in this case different than when we ran the 1 way ANOVA, since there is only 1 df for the numerator. That is, while the F_{crit} for the 1 way ANOVA with 2 and 12 df is 3.89, the F_{crit} for these post hoc comparisons has 1 and 12 df and is thus, 4.75. Thus, applying the formula to the three possible pairwise comparisons reveals:

$$\mathbf{F}_{1x2} = \frac{(43.6 - 51.8)^2}{28.3\left(\frac{1}{5} + \frac{1}{5}\right)} = \frac{(-8.2)^2}{28.3(.4)} = \frac{67.24}{11.32} = 5.94$$

$$\mathbf{F}_{1x3} = \frac{(43.6 - 55)^2}{28.3\left(\frac{1}{5} + \frac{1}{5}\right)} = \frac{(-11.4)^2}{28.3(.4)} = \frac{129.96}{11.32} = 11.48$$

$$\mathbf{F}_{2x3} = \frac{(51.8 - 55)^2}{28.3\left(\frac{1}{5} + \frac{1}{5}\right)} = \frac{(-3.2)^2}{28.3(.4)} = \frac{10.24}{11.32} = 0.904$$

Both the 1-pill and 5-pill groups showed improved performance when compared to the placebo group. However, the two drug groups did not differ from each other.

2) 1. Research Question
Does sleep deprivation affect the ability to detect moving objects?

2. Hypotheses

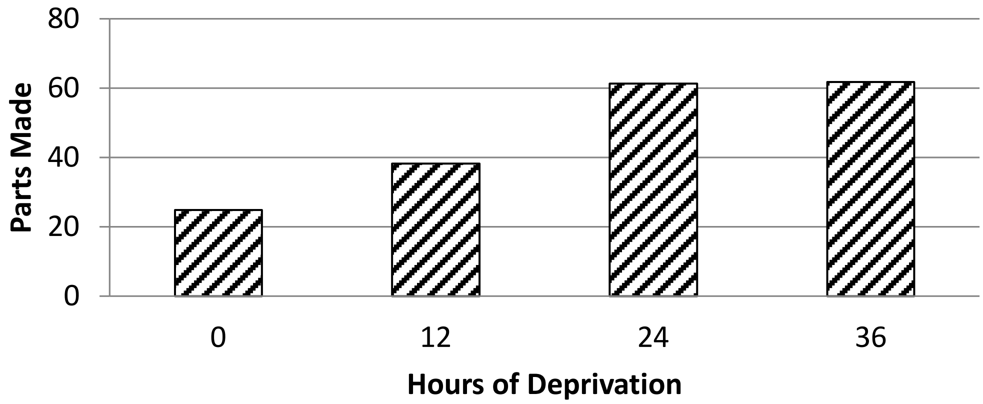
	Symbols	Words
H_O	$\mu_1=\mu_2=\mu_3=\mu_4$	Deprivation does not affect the detection ability.
H_A	Not H_o	Deprivation does affect the detection ability.

3. Assumptions
1. H_o
2. Subjects are sampled randomly.
3. Population distribution of the DV is normal in shape.
4. Groups are independent.
5. Population variances are homogenous.

4. Decision rules
Given 4 groups with 5, 5, 5, and 5 subjects, respectively, we have (4-1=) 3 df for the between groups variance estimate and (4+4+4+4) = 16 df for the within groups variance estimate. Alpha level of .05, the critical value of F is 3.24. If $F_{obs} \geq F_{crit}$, reject H_o, otherwise do not reject H_o.

5. Computation
We will create a table that has the quantities we need.

	0 hr	**X^2**	**12 hr**	**X^2**	**24 hr**	**X^2**	**36 hr**	**X^2**	
	36	1296	38	1444	46	2116	76	5776	
	21	441	45	2025	74	5476	66	4356	
	20	400	46	2116	67	4489	62	3844	
	26	676	22	484	61	3721	44	1936	
	21	441	40	1600	59	3481	61	3721	
T_j	124		191		307		309		931 = **T**
n_j	5		5		5		5		20 = **N**
$\bar{X}_j$	24.8		38.2		61.4		61.8		
$\sum_{i=1}^{n_j} X_{ij}^2$		3254		7669		19283		19633	49839 = **II**
$\frac{T_j^2}{n_j}$	3075.2		7296.2		18849.8		19096.2		48317.4 = **III**



I. $$\frac{T^2}{N} = \frac{931^2}{20} = \frac{866,761}{20} = 43338.05$$

II. $$\sum\left(\sum_{i=1}^{n_j} X_{ij}^2\right) = 49839$$

III. $$\sum\left(\frac{T_j^2}{n_j}\right) = 48317.4$$

$$\mathbf{SS_B} = III - I = 48317.4 - 43338.05 = 4979.35$$

$$\mathbf{SS_w} = II - III = 49839 - 48317.4 = 1521.6$$

$$\mathbf{SS_T} = II - I = 49839 - 43338.05 = 6500.95$$

$$\mathbf{MS_B} = \frac{SS_B}{df_B} = \frac{4979.35}{3} = 1659.78$$

$$\mathbf{MS_W} = \frac{SS_W}{df_W} = \frac{1521.6}{16} = 95.1$$

$$\mathbf{F} = \frac{MS_B}{MS_W} = \frac{1659.78}{95.1} = 17.45$$

Source	SS	df	MS	F	p
Between	4979.35	3	1659.78	17.45	< .05
Within	1521.6	16	95.1		
Total	6500.95	19			

6. **Decision**

Since F_{obs} (17.45) is > F_{crit} (3.24), reject H_o and conclude that the more sleep deprivation yields more failures in identifying moving objects. Since the omnibus F is significant, we need to perform post hoc comparisons to localize the effect. The F_{crit} for these comparisons uses 1 and 16 df and is thus equal to 4.49

$$\mathbf{F}_{\mathbf{Comp}} = \frac{(\overline{X}_1 - \overline{X}_2)^2}{MS_w\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}$$

$$\mathbf{F}_{1x2} = \frac{(24.8 - 38.2)^2}{95.1\left(\frac{1}{5} + \frac{1}{5}\right)} = \frac{(-13.4)^2}{95.1(.4)} = \frac{179.56}{38.04} = 4.72$$

$$\mathbf{F}_{1x3} = \frac{(24.8 - 61.4)^2}{95.1\left(\frac{1}{5} + \frac{1}{5}\right)} = \frac{(-36.6)^2}{95.1(.4)} = \frac{1339.56}{38.04} = 35.21$$

$$\mathbf{F}_{1x4} = \frac{(24.8 - 61.8)^2}{95.1\left(\frac{1}{5} + \frac{1}{5}\right)} = \frac{(-37)^2}{95.1(.4)} = \frac{1369}{38.04} = 35.99$$

$$\mathbf{F}_{2x3} = \frac{(38.2 - 61.4)^2}{95.1\left(\frac{1}{5} + \frac{1}{5}\right)} = \frac{(-23.2)^2}{95.1(.4)} = \frac{538.24}{38.04} = 14.15$$

$$\mathbf{F}_{2x4} = \frac{(38.2 - 61.8)^2}{95.1\left(\frac{1}{5} + \frac{1}{5}\right)} = \frac{(-23.6)^2}{95.1(.4)} = \frac{556.96}{38.04} = 14.64$$

$$\mathbf{F}_{3x4} = \frac{(61.4 - 61.8)^2}{95.1\left(\frac{1}{5} + \frac{1}{5}\right)} = \frac{(-0.4)^2}{95.1(.4)} = \frac{0.16}{38.04} = 0.004$$

All 6 comparisons are significant except one. In other words, sleep deprivation yields more failures in identifying objects except when comparing the 24 hour deprivation group to the 36 hour deprivation group.

3)

Source	**SS**	**df**	**MS**	**F**	**p**
Between	132.00	3	44.00	.44	> .05
Within	1596.00	16	99.75		
Total	1728.00	19			

Since there $df_{between}$=3, there must be 4 groups. Since df_{within}=16 and there is 4 groups, there must be 5 subjects per group.

4) With three groups (Groups 1, 2 & 3), the following 6 comparisons are possible:

Simple	Complex
1 vs 2	(1+2) vs 3
1 vs 3	1 vs (2+3)
2 vs 3	(1+3) vs 2

Simple – occurs between pairs of means (illustrated by the left hand column)
Complex – involve more than two means (illustrated by the right hand column)

5) c – increases F, which is good

6) a – decreases F