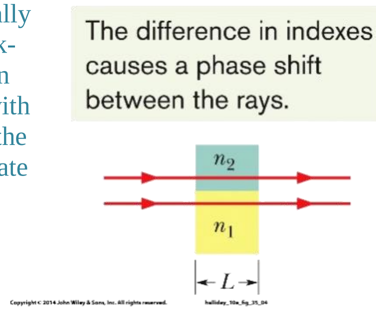
Due Monday 12/10/18 at 11:00am          Name ___________________________________

# Homework Chapter 35: Interference

**35.03** In Fig. 35-4, assume that two waves of light in air, of wavelength 400 nm, are initially in phase. One travels through a glass layer of index of refraction $n_1 = 1.60$ and thickness $L$. The other travels through an equally thick plastic layer of index of refraction $n_2 = 1.50$. (a) What is the smallest value $L$ should have if the waves are to end up with a phase difference of 5.65 rad? (b) If the waves arrive at some common point with the same amplitude, is their interference fully constructive, fully destructive, intermediate but closer to fully constructive, or intermediate but closer to fully destructive?




3. **THINK** The wavelength of light in a medium depends on the index of refraction of the medium. The nature of the interference, whether constructive or destructive, depends on the phase difference of the two waves.

**EXPRESS** We take the phases of both waves to be zero at the front surfaces of the layers. The phase of the first wave at the back surface of the glass is given by $\phi_1 = k_1L - \omega t$, where $k_1$ (= $2\pi/\lambda_1$) is the angular wave number and $\lambda_1$ is the wavelength in glass. Similarly, the phase of the second wave at the back surface of the plastic is given by $\phi_2 = k_2L - \omega t$, where $k_2$ (= $2\pi/\lambda_2$) is the angular wave number and $\lambda_2$ is the wavelength in plastic. The angular frequencies are the same since the waves have the same wavelength in air and the frequency of a wave does not change when the wave enters another medium. The phase difference is

$$\phi_1 - \phi_2 = (k_1 - k_2)L = 2\pi\left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right)L.$$

Now, $\lambda_1 = \lambda_{air}/n_1$, where $\lambda_{air}$ is the wavelength in air and $n_1$ is the index of refraction of the glass. Similarly, $\lambda_2 = \lambda_{air}/n_2$, where $n_2$ is the index of refraction of the plastic. This means that the phase difference is

$$\phi_1 - \phi_2 = \frac{2\pi}{\lambda_{air}}(n_1 - n_2)L.$$

**ANALYZE** (a) The value of $L$ that makes this 5.65 rad is

$$L = \frac{(\phi_1 - \phi_2)\lambda_{air}}{2\pi(n_1 - n_2)} = \frac{5.65(400\times10^{-9}\,\text{m})}{2\pi(1.60 - 1.50)} = 3.60\times10^{-6}\,\text{m}.$$

(b) A phase difference of 5.65 rad is less than $2\pi$ rad = 6.28 rad, the phase difference for completely constructive interference, but greater than $\pi$ rad (= 3.14 rad), the phase difference for completely destructive interference. The interference is, therefore, intermediate, neither completely constructive nor completely destructive. It is, however, closer to completely constructive than to completely destructive.

5.65 rad = $1.80\pi$ = 324° = −36°
(closer to completely constructive [0° or 360°] than to completely destructive [180°])

**LEARN** The phase difference of two light waves can change when they travel through different materials having different indices of refraction.

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**35.19** Suppose that Young's experiment is performed with blue-green light of wavelength 500 nm. The slits are 1.20 mm apart, and the viewing screen is 5.40 m from the slits. How far apart are the bright fringes near the center of the interference pattern?

19. **THINK** The condition for a maximum in the two-slit interference pattern is $d\sin\theta = m\lambda$, where $d$ is the slit separation, $\lambda$ is the wavelength, $m$ is an integer, and $\theta$ is the angle made by the interfering rays with the forward direction.

**EXPRESS** If $\theta$ is small, $\sin\theta$ may be approximated by $\theta$ in radians. Then, $\theta = m\lambda/d$, and the angular separation of adjacent maxima, one associated with the integer $m$ and the other associated with the integer $m + 1$, is given by $\Delta\theta = \lambda/d$. The separation on a screen a distance $D$ away is given by

$$\Delta y = D\,\Delta\theta = \lambda D/d.$$

**ANALYZE** Thus,

$$\Delta y = \frac{(500\times10^{-9}\,\text{m})(5.40\,\text{m})}{1.20\times10^{-3}\,\text{m}} = 2.25\times10^{-3}\,\text{m} = 2.25\,\text{mm}.$$

**LEARN** For small $\theta$, the spacing is nearly uniform. However, away from the center of the pattern, $\theta$ increases and the spacing gets larger.

**35.21** In a double-slit experiment, the distance between slits is 5.0 mm and the slits are 1.0 m from the screen. Two interference patterns can be seen on the screen: one due to light of wavelength 480 nm, and the other due to light of wavelength 600 nm. What is the separation on the screen between the third-order (*m* = 3) bright fringes of the two interference patterns?

21. The maxima of a two-slit interference pattern are at angles $\theta$ given by $d \sin\theta = m\lambda$, where $d$ is the slit separation, $\lambda$ is the wavelength, and $m$ is an integer. If $\theta$ is small, $\sin\theta$ may be replaced by $\theta$ in radians. Then, $d\theta = m\lambda$. The angular separation of two maxima associated with different wavelengths but the same value of $m$ is

$$\Delta\theta = (m/d)(\lambda_2 - \lambda_1),$$

and their separation on a screen a distance $D$ away is

$$\Delta y = D\tan\Delta\theta \approx D\,\Delta\theta = \left[\frac{mD}{d}\right](\lambda_2 - \lambda_1)$$
$$= \left[\frac{3(1.0\,\text{m})}{5.0\times10^{-3}\,\text{m}}\right]\left(600\times10^{-9}\,\text{m} - 480\times10^{-9}\,\text{m}\right) = 7.2\times10^{-5}\,\text{m} = 0.0720\ \text{mm} = 72.0\ \mu\text{m}$$

The small angle approximation $\tan\Delta\theta \approx \Delta\theta$ (in radians) is made.

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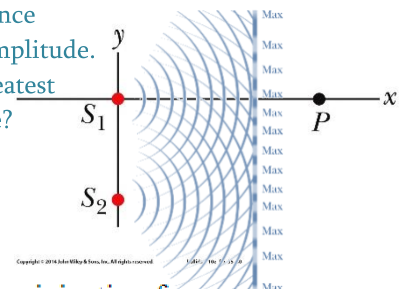
**35.25** In Fig. 35-40, two isotropic point sources of light ($S_1$ and $S_2$) are separated by distance 2.70 $\mu$m along a $y$ axis and emit in phase at wavelength 900 nm and at the same amplitude. A light detector is located at point $P$ at coordinate $x_P$ on the $x$ axis. What is the greatest value of $x_P$ at which the detected light is minimum due to destructive interference?




25. Let the distance in question be $x$. The path difference (between rays originating from $S_1$ and $S_2$ and arriving at points on the $x > 0$ axis) is

$$\sqrt{d^2 + x^2} - x = \left(m + \frac{1}{2}\right)\lambda,$$

Skipped algebra: add $x$ to both sides and square:

$$d^2 + x^2 = \left[\left(m+\tfrac{1}{2}\right)\lambda + x\right]^2$$
$$d^2 + x^2 = \left(m+\tfrac{1}{2}\right)^2\lambda^2 + 2\left(m+\tfrac{1}{2}\right)\lambda x + x^2$$

where we are requiring destructive interference (half-integer wavelength phase differences) and $m = 0, 1, 2, \ldots$. After some algebraic steps, we solve for the distance in terms of $m$:

Subtract $x^2$ from both sides and solve for $x$:

$$\frac{d^2 - \left(m+\frac{1}{2}\right)^2\lambda^2}{2\left(m+\frac{1}{2}\right)\lambda} = x = \frac{d^2}{2\left(m+\frac{1}{2}\right)\lambda} - \frac{\left(m+\frac{1}{2}\right)\lambda}{2}$$

$$x = \frac{d^2}{(2m+1)\lambda} - \frac{(2m+1)\lambda}{4}.$$

Simplify: $x = \dfrac{d^2}{(2m+1)\lambda} - \dfrac{(2m+1)\lambda}{4}$

Solutions:

| $m$: | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| $x$ ($\mu$m): | 7.88 | 2.03 | 0.50 | −0.42 |

To obtain the largest value of $x$, we set $m = 0$:

$$x_0 = \frac{d^2}{\lambda} - \frac{\lambda}{4} = \frac{(3.00\lambda)^2}{\lambda} - \frac{\lambda}{4} = 8.75\lambda = 8.75(900\ \text{nm}) = 7.88\times10^3\ \text{nm} = 7.88\,\mu\text{m}.$$

**35.35** We wish to coat flat glass ($n$ = 1.50) with a transparent material ($n$ = 1.25) so that reflection of light at wavelength 600 nm is eliminated by interference. What minimum thickness can the coating have to do this?

35. **THINK** For complete destructive interference, we want the waves reflected from the front and back of the coating to differ in phase by an odd multiple of $\pi$ rad.

**EXPRESS** Each wave is incident on a medium of higher index of refraction from a medium of lower index, so both suffer phase changes of $\pi$ rad on reflection. If $L$ is the thickness of the coating, the wave reflected from the back surface travels a distance $2L$ farther than the wave reflected from the front. The phase difference is $2L(2\pi/\lambda_c)$, where $\lambda_c$ is the wavelength in the coating. If $n$ is the index of refraction of the coating, $\lambda_c = \lambda/n$, where $\lambda$ is the wavelength in vacuum, and the phase difference is $2nL(2\pi/\lambda)$. We solve

$$2nL\left(\frac{2\pi}{\lambda}\right) = (2m+1)\pi$$

for $L$. Here $m$ is an integer. The result is $L = \frac{(2m+1)\lambda}{4n}$.

**ANALYZE** To find the least thickness for which destructive interference occurs, we take $m = 0$. Then,

$$L = \frac{\lambda}{4n} = \frac{600\times10^{-9}\,\text{m}}{4(1.25)} = 1.20\times10^{-7}\,\text{m.} = 120\text{ nm}$$

**LEARN** A light ray reflected by a material changes phase by $\pi$ rad (or 180°) if the refractive index of the material is greater than that of the medium in which the light is traveling.

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**35.55** A disabled tanker leaks kerosene ($n$ = 1.20) into the Persian Gulf, creating a large slick on top of the water ($n$ = 1.30). (a) If you are looking straight down from an airplane, while the Sun is overhead, at a region of the slick where its thickness is 460 nm, for which wavelength(s) of visible light is the reflection brightest because of constructive interference? (b) If you are scuba diving directly under this same region of the slick, for which wavelength(s) of visible light is the transmitted intensity strongest?

55. **THINK** The index of refraction of oil is greater than that of the air, but smaller than that of the water.

**EXPRESS** Let the indices of refraction of the air, oil and water be $n_1$, $n_2$, and $n_3$, respectively. Since $n_1 < n_2$ and $n_2 < n_3$, there is a phase change of $\pi$ rad from both surfaces. Since the second wave travels an additional distance of $2L$, the phase difference is

$$\phi = \frac{2\pi}{\lambda_2}(2L)$$

where $\lambda_2 = \lambda / n_2$ is the wavelength in the oil. The condition for constructive interference is

$$\frac{2\pi}{\lambda_2}(2L) = 2m\pi,$$

or

$$2L = m\frac{\lambda}{n_2}, \quad m = 0, 1, 2, \ldots$$

**ANALYZE** (a) For $m = 1, 2, \ldots$, maximum reflection occurs for wavelengths

$$\lambda = \frac{2n_2L}{m} = \frac{2(1.20)(460\,\text{nm})}{m} = 1104\,\text{nm}\,,\ 552\,\text{nm},\ 368\,\text{nm}\ldots$$

We note that only the 552 nm wavelength falls within the visible light range.

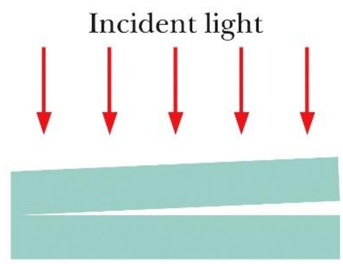
(b) Maximum transmission into the water occurs for wavelengths for which reflection is a minimum. The condition for such destructive interference is given by

$$2L = \left(m+\frac{1}{2}\right)\frac{\lambda}{n_2} \Rightarrow \lambda = \frac{4n_2L}{2m+1}$$

which yields $\lambda$ = 2208 nm, 736 nm, 442 nm … for the different values of $m$. We note that only the 442 nm wavelength (blue) is in the visible range, though we might expect some red contribution since the 736 nm is very close to the visible range.

**LEARN** A light ray reflected by a material changes phase by $\pi$ rad (or 180°) if the refractive index of the material is greater than that of the medium in which the light is traveling. Otherwise, there is no phase change. Note that refraction at an interface does not cause a phase shift.

**35.73** In Fig. 35-45, a broad beam of light of wavelength 683 nm is sent directly downward through the top plate of a pair of glass plates. The plates are 120 mm long, touch at the left end, and are separated by 48.0 $\mu$m at the right end. The air between the plates acts as a thin film. How many bright fringes will be seen by an observer looking down through the top plate?




73. **THINK** A light ray reflected by a material changes phase by $\pi$ rad (or 180°) if the refractive index of the material is greater than that of the medium in which the light is traveling.

**EXPRESS** Consider the interference of waves reflected from the top and bottom surfaces of the air film. The wave reflected from the upper surface does not change phase on reflection but the wave reflected from the bottom surface changes phase by $\pi$ rad. At a place where the thickness of the air film is $L$, the condition for fully constructive interference is $2L = (m+\frac{1}{2})\lambda$ where $\lambda$ (= 683 nm) is the wavelength and $m$ is an integer.

**ANALYZE** For $L = 48\ \mu$m, we find the value of m to be

$$m = \frac{2L}{\lambda} - \frac{1}{2} = \frac{2(4.80\times10^{-5}\text{m})}{683\times10^{-9}\text{m}} - \frac{1}{2} = 140.$$

At the thin end of the air film, there is a bright fringe. It is associated with $m = 0$. There are, therefore, 140 bright fringes in all.

**LEARN** The number of bright fringes increases with $L$, but decreases with $\lambda$.

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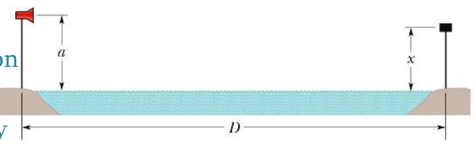
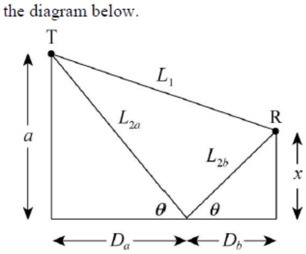
**35.89** In Fig. 35-53, a microwave transmitter at height $a$ above the water level of a wide lake transmits microwaves of wavelength $\lambda$ toward a receiver on the opposite shore, a distance $x$ above the water level. The microwaves reflecting from the water interfere with the microwaves arriving directly from the transmitter. Assuming that the lake width $D$ is much greater than $a$ and $x$, and that $\lambda \geq a$, find an expression that gives the values of $x$ for which the signal at the receiver is maximum. (*Hint:* Does the reflection cause a phase change?)




89. **THINK** Since the index of refraction of water is greater than that of air, the wave that is reflected from the water surface suffers a phase change of $\pi$ rad on reflection.

**EXPRESS** Suppose the wave that goes directly to the receiver travels a distance $L_1$ and the reflected wave travels a distance $L_2$. The last wave suffers a phase change on reflection of half a wavelength since water has higher refractive index than air. To obtain constructive interference at the receiver, the difference $L_2 - L_1$ must be an odd multiple of a half wavelength.

**ANALYZE** Consider the diagram below.




The right triangle on the left, formed by the vertical line from the water to the transmitter T, the ray incident on the water, and the water line, gives $D_a = a/\tan\theta$. The right triangle on the right, formed by the vertical line from the water to the receiver $R$, the reflected ray, and the water line leads to $D_b = x/\tan\theta$. Since $D_a + D_b = D$,

$$\tan\theta = \frac{a+x}{D}.$$

We use the identity $\sin^2\theta = \tan^2\theta/(1+\tan^2\theta)$ to show that

$$\sin\theta = (a+x)/\sqrt{D^2+(a+x)^2}.$$

This means

$$L_{2a} = \frac{a}{\sin\theta} = \frac{a\sqrt{D^2+(a+x)^2}}{a+x}$$

and

$$L_{2b} = \frac{x}{\sin\theta} = \frac{x\sqrt{D^2+(a+x)^2}}{a+x}.$$

Therefore,

$$L_2 = L_{2a} + L_{2b} = \frac{(a+x)\sqrt{D^2+(a+x)^2}}{a+x} = \sqrt{D^2+(a+x)^2}.$$

Using the binomial theorem, with $D^2$ large and $a^2 + x^2$ small, we approximate this expression: $L_2 \approx D + (a+x)^2/2D$. The distance traveled by the direct wave is $L_1 = \sqrt{D^2+(a-x)^2}$. Using the binomial theorem, we approximate this expression: $L_1 \approx D + (a-x)^2/2D$. Thus,

$$L_2 - L_1 \approx D + \frac{a^2+2ax+x^2}{2D} - D - \frac{a^2-2ax+x^2}{2D} = \frac{2ax}{D}.$$

Setting this equal to $(m+\frac{1}{2})\lambda$, where $m$ is zero or a positive integer, we find $x = (m+\frac{1}{2})(\lambda D/2a)$.

Notice the result is the opposite of the two-slit pattern (due to the phase shift) with $d = 2a$:

$$(2a)\sin\theta_{\text{bright}} = (m+\tfrac{1}{2})\lambda \Rightarrow (2a)\left(\frac{x}{D}\right) = (m+\tfrac{1}{2})\lambda$$

**LEARN** Similarly, the condition for destructive interference is

$$L_2 - L_1 \approx \frac{2ax}{D} = m\lambda,$$

or

$$x = m\frac{\lambda D}{2a}, \quad m = 0, 1, 2, \ldots$$