Due Monday 9/17/18 at 11:00am Name ____________________________________

Homework Chapter 21: Coulomb's Law

21.03 What must be the distance between point charge $q_1 = 26.0\ \mu C$ and point charge $q_2 = -47.0\ \mu C$ for the electrostatic force between them to have a magnitude of 5.70 N?

3. **THINK** The magnitude of the electrostatic force between two charges q_1 and q_2 separated by a distance r is given by Coulomb's law.

EXPRESS Equation 21-1 gives Coulomb's law, $F = k\frac{|q_1||q_2|}{r^2}$, which can be used to solve for the distance:

$$r = \sqrt{\frac{k|q_1||q_2|}{F}}.$$

ANALYZE With $F = 5.70\ \mathrm{N}$, $q_1 = 2.60 \times 10^{-6}\ \mathrm{C}$ and $q_2 = -47.0 \times 10^{-6}\ \mathrm{C}$, the distance between the two charges is

$$r = \sqrt{\frac{k|q_1||q_2|}{F}} = \sqrt{\frac{\left(8.99\times10^{9}\,\mathrm{N\cdot m^2/C^2}\right)\left(26.0\times10^{-6}\mathrm{C}\right)\left(47.0\times10^{-6}\mathrm{C}\right)}{5.70\mathrm{N}}} = 1.39\ \mathrm{m}.$$

LEARN The electrostatic force between two charges falls as $1/r^2$. The same inverse-square nature is also seen in the gravitational force between two masses.

21.04 In the return stroke of a typical lightning bolt, a current of 2.5×10^4 A exists for 20 μs. How much charge is transferred in this event?

4. The unit ampere is discussed in Section 21-4. Using i for current, the charge transferred is

$$q = it = \left(2.5\times10^{4}\,\mathrm{A}\right)\left(20\times10^{-6}\,\mathrm{s}\right) = 0.50\ \mathrm{C}.$$

21.09 Two identical conducting spheres, fixed in place, attract each other with an electrostatic force of 0.108 N when their center-to-center separation is 50.0 cm. The spheres are then connected by a thin conducting wire. When the wire is removed, the spheres repel each other with an electrostatic force of 0.0360 N. Of the initial charges on the spheres, with a positive net charge, what was (a) the negative charge on one of them and (b) the positive charge on the other?

9. **THINK** Since opposite charges attract, the initial charge configurations must be of opposite signs. Similarly, since like charges repel, the final charge configurations must carry the same sign.

EXPRESS We assume that the spheres are far apart. Then the charge distribution on each of them is spherically symmetric and Coulomb's law can be used. Let q_1 and q_2 be the original charges. We choose the coordinate system so the force on q_2 is positive if it is repelled by q_1. Then the force on q_2 is

$$F_a = -\frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r^2} = -k\frac{q_1q_2}{r^2}$$

where $k = 1/4\pi\varepsilon_0 = 8.99\times10^9\,\mathrm{N\cdot m^2/C^2}$ and $r = 0.500$ m. The negative sign indicates that the spheres attract each other. After the wire is connected, the spheres, being identical, acquire the same charge. Since charge is conserved, the total charge is the same as it was originally. This means the charge on each sphere is $(q_1 + q_2)/2$. The force is now repulsive and is given by

$$F_b = \frac{1}{4\pi\varepsilon_0}\frac{\left(\frac{q_1+q_2}{2}\right)\left(\frac{q_1+q_2}{2}\right)}{r^2} = k\frac{(q_1+q_2)^2}{4r^2}.$$

We solve the two force equations simultaneously for q_1 and q_2.

ANALYZE The first equation gives the product

$$q_1q_2 = -\frac{r^2F_a}{k} = -\frac{(0.500\,\mathrm{m})^2(0.108\,\mathrm{N})}{8.99\times10^9\,\mathrm{N\cdot m^2/C^2}} = -3.00\times10^{-12}\,\mathrm{C^2},$$

and the second gives the sum

$$q_1 + q_2 = 2r\sqrt{\frac{F_b}{k}} = 2(0.500\,\mathrm{m})\sqrt{\frac{0.0360\,\mathrm{N}}{8.99\times10^9\,\mathrm{N\cdot m^2/C^2}}} = 2.00\times10^{-6}\,\mathrm{C}$$

where we have taken the positive root (which amounts to assuming $q_1 + q_2 \geq 0$). Thus, the product result provides the relation

$$q_2 = \frac{-\left(3.00\times10^{-12}\,\mathrm{C^2}\right)}{q_1}$$

which we substitute into the sum result, producing

$$q_1 - \frac{3.00\times10^{-12}\,\mathrm{C^2}}{q_1} = 2.00\times10^{-6}\,\mathrm{C}.$$

21.09 (continued)

Multiplying by q_1 and rearranging, we obtain a quadratic equation

$$q_1^2 - (2.00 \times 10^{-6}\,\text{C})q_1 - 3.00 \times 10^{-12}\,\text{C}^2 = 0.$$

The solutions are

$$q_1 = \frac{2.00 \times 10^{-6}\,\text{C} \pm \sqrt{(-2.00 \times 10^{-6}\,\text{C})^2 - 4(-3.00 \times 10^{-12}\,\text{C}^2)}}{2}.$$

If the positive sign is used, $q_1 = 3.00 \times 10^{-6}$ C, and if the negative sign is used, $q_1 = -1.00 \times 10^{-6}\,\text{C}$.

(a) Using $q_2 = (-3.00 \times 10^{-12})/q_1$ with $q_1 = 3.00 \times 10^{-6}$ C, we get $q_2 = -1.00 \times 10^{-6}\,\text{C}$.

(b) If we instead work with the $q_1 = -1.00 \times 10^{-6}$ C root, then we find $q_2 = 3.00 \times 10^{-6}\,\text{C}$.

LEARN Note that since the spheres are identical, the solutions are essentially the same: one sphere originally had charge -1.00×10^{-6} C and the other had charge $+3.00 \times 10^{-6}$ C. What happens if we had not made the assumption, above, that $q_1 + q_2 \geq 0$? If the signs of the charges were reversed (so $q_1 + q_2 < 0$), then the forces remain the same, so a charge of $+1.00 \times 10^{-6}$ C on one sphere and a charge of -3.00×10^{-6} C on the other also satisfies the conditions of the problem.

(Solutions continued next page)

21.57 We know that the negative charge on the electron and the positive charge on the proton are equal. Suppose, however, that these magnitudes differ from each other by 0.00010%. With what force would two copper coins, placed 1.0 m apart, repel each other? Assume that each coin contains 3×10^{22} copper atoms. (Hint: A neutral copper atom contains 29 protons and 29 electrons.) What do you conclude?

57. If the relative difference between the proton and electron charges (in absolute value) were

$$\frac{q_p - |q_e|}{e} = 0.0000010$$

then the actual difference would be $q_p - |q_e| = 1.6\times10^{-25}$ C. Amplified by a factor of 29 × 3 × 10^{22} as indicated in the problem, this amounts to a deviation from perfect neutrality of

$$\Delta q = \left(29\times3\times10^{22}\right)\left(1.6\times10^{-25}\text{ C}\right) = 0.14\text{ C}$$

in a copper penny. Two such pennies, at r = 1.0 m, would therefore experience a very large force. Equation 21-1 gives

$$F = k\frac{(\Delta q)^2}{r^2} = 1.7\times10^{8}\text{ N}.$$

$\boxed{\text{17,000 metric tons!}}$

21.73 In an early model of the hydrogen atom (the *Bohr model*), the electron orbits the proton in uniformly circular motion. The radius of the circle is restricted (*quantized*) to certain values given by

$r = n^2a_0$, for n = 1, 2, 3, … ,

where a_0 = 52.92 pm. What is the speed of the electron if it orbits in (a) the smallest allowed orbit and (b) the second smallest orbit? (c) If the electron moves to larger orbits, does its speed increase, decrease, or stay the same?

73. (a) The Coulomb force between the electron and the proton provides the centripetal force that keeps the electron in circular orbit about the proton:

$$\frac{k|e|^2}{r^2} = \frac{m_e v^2}{r}$$

The smallest orbital radius is $r_1 = a_0 = 52.9\times10^{-12}$ m. The corresponding speed of the electron is

$$v_1 = \sqrt{\frac{k|e|^2}{m_e r_1}} = \sqrt{\frac{k|e|^2}{m_e a_0}} = \sqrt{\frac{(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(1.6\times10^{-19}\text{ C})^2}{(9.11\times10^{-31}\text{ kg})(52.9\times10^{-12}\text{m})}}$$
$$= 2.19\times10^6\text{ m/s}.$$

(b) The radius of the second smallest orbit is $r_2 = (2)^2 a_0 = 4a_0$. Thus, the speed of the electron is

$$v_2 = \sqrt{\frac{k|e|^2}{m_e r_2}} = \sqrt{\frac{k|e|^2}{m_e(4a_0)}} = \frac{1}{2}v_1 = \frac{1}{2}(2.19\times10^6\text{ m/s})$$
$$= 1.09\times10^6\text{ m/s}.$$

(c) Since the speed is inversely proportional to $r^{1/2}$, the speed of the electron will decrease if it moves to larger orbits.

Homework Chapter 22: Electric Fields

22.05. A charged particle produces an electric field with a magnitude of 2.0 N/C at a point that is 50 cm away from the particle. What is the magnitude of the particle's charge?

5. **THINK** The magnitude of the electric field produced by a point charge q is given by $E = |q|/4\pi\varepsilon_0 r^2$, where r is the distance from the charge to the point where the field has magnitude E.

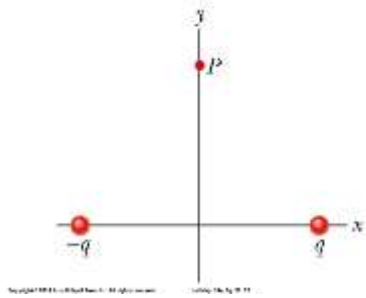
EXPRESS From $E = |q|/4\pi\varepsilon_0 r^2$, the magnitude of the charge is $|q| = 4\pi\varepsilon_0 r^2 E$.

ANALYZE With $E = 2.0$ N/C at $r = 50$ cm $= 0.50$ m, we obtain

$$|q| = 4\pi\varepsilon_0 r^2 E = \frac{(0.50\,\text{m})^2(2.0\,\text{N/C})}{8.99\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2} = 5.6\times10^{-11}\,\text{C}.$$

LEARN To determine the sign of the charge, we would need to know the direction of the field. The field lines extend away from a positive charge and toward a negative charge.

22.09. Figure 22-37 shows two charged particles on an x axis: $-q = -3.20 \times 10^{-19}$ C at $x = -3.00$ m and $q = 3.20 \times 10^{-19}$ C at $x = +3.00$ m. What are the (a) magnitude and (b) direction (relative to the positive direction of the x axis) of the net electric field produced at point P at $y = 4.00$ m?



9. (a) The vertical components of the individual fields (due to the two charges) cancel, by symmetry. Using $d = 3.00$ m and $y = 4.00$ m, the horizontal components (both pointing to the $-x$ direction) add to give a magnitude of

$$E_{x,\text{net}} = \frac{2|q|d}{4\pi\varepsilon_0(d^2+y^2)^{3/2}} = \frac{2(8.99\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2)(3.20\times10^{-19}\,\text{C})(3.00\,\text{m})}{[(3.00\,\text{m})^2+(4.00\,\text{m})^2]^{3/2}}.$$
$$= 1.38\times10^{-10}\,\text{N/C}.$$

(b) The net electric field points in the $-x$ direction, or 180° counterclockwise from the $+x$ axis.

22.11. Two charged particles are fixed to an x axis: Particle 1 of charge $q_1 = 2.1 \times 10^{-8}$ C is at position $x = 20$ cm and particle 2 of charge $q_2 = -4.00q_1$ is at position $x = 70$ cm. At what coordinate on the axis (other than at infinity) is the net electric field produced by the two particles equal to zero?

11. **THINK** Our system consists of two point charges of opposite signs fixed to the x axis. Since the net electric field at a point is the vector sum of the electric fields of individual charges, there exists a location where the net field is zero.

EXPRESS At points between the charges, the individual electric fields are in the same direction and do not cancel. Since charge $q_2 = -4.00\, q_1$ located at $x_2 = 70$ cm has a greater magnitude than $q_1 = 2.1 \times 10^{-8}$ C located at $x_1 = 20$ cm, a point of zero field must be closer to q_1 than to q_2. It must be to the left of q_1.

Let x be the coordinate of P, the point where the field vanishes. Then, the total electric field at P is given by

$$E = \frac{1}{4\pi\varepsilon_0}\left(\frac{|q_2|}{(x-x_2)^2} - \frac{|q_1|}{(x-x_1)^2}\right).$$

ANALYZE If the field is to vanish, then

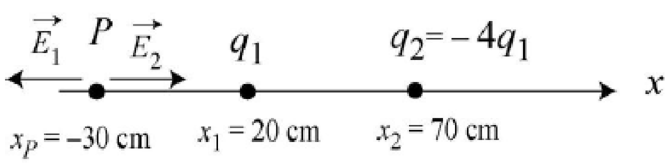
$$\frac{|q_2|}{(x-x_2)^2} = \frac{|q_1|}{(x-x_1)^2} \quad \Rightarrow \quad \frac{|q_2|}{|q_1|} = \frac{(x-x_2)^2}{(x-x_1)^2}.$$

Taking the square root of both sides, noting that $|q_2|/|q_1| = 4$, we obtain

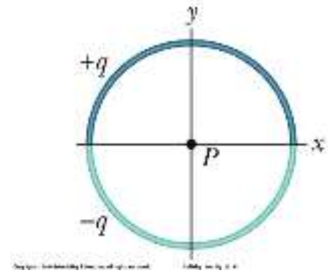
$$\frac{x - 70\text{ cm}}{x - 20\text{ cm}} = \pm 2.0\,.$$

Choosing -2.0 for consistency, the value of x is found to be $x = -30$ cm.

LEARN The results are depicted in the figure below. At P, the field $\vec{E}_1$ due to q_1 points to the left, while the field $\vec{E}_2$ due to q_2 points to the right. Since $|\vec{E}_1| = |\vec{E}_2|$, the net field at P is zero.



22.27. In Fig. 22-51, two curved plastic rods, one of charge $+q$ and the other of charge $-q$, form a circle of radius R = 8.50 cm in an xy plane. The x axis passes through both of the connecting points, and the charge is distributed uniformly on both rods. If q = 15.0 pC, what are the (a) magnitude and (b) direction (relative to the positive direction of the x axis) of the electric field $\vec{\mathbf{E}}$ produced at P, the center of the circle?



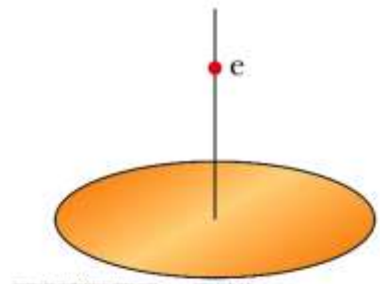
27. From symmetry, we see that the net field at P is twice the field caused by the upper semicircular charge $+q = \lambda(\pi R)$ (and that it points downward). Adapting the steps leading to Eq. 22-21, we find

$$\vec{E}_{\text{net}} = 2\left(-\hat{\mathrm{j}}\right)\frac{\lambda}{4\pi\varepsilon_0 R}\sin\theta\Big|_{-90°}^{90°} = -\left(\frac{q}{\varepsilon_0 \pi^2 R^2}\right)\hat{\mathrm{j}}.$$

(a) With $R = 8.50 \times 10^{-2}$ m and $q = 1.50 \times 10^{-8}$ C, $|\vec{E}_{\text{net}}| = 23.8$ N/C.

(b) The net electric field $\vec{E}_{\text{net}}$ points in the $-\hat{\mathrm{j}}$ direction, or $-90°$ counterclockwise from the $+x$ axis.

22.48. In Fig. 22-59, an electron (e) is to be released from rest on the central axis of a uniformly charged disk of radius R. The surface charge density on the disk is $+4.00\ \mu\text{C/m}^2$. What is the magnitude of the electron's initial acceleration if it is released at a distance (a) R, (b) R/100, and (c) R/1000 from the center of the disk? (d) Why does the acceleration magnitude increase only slightly as the release point is moved closer to the disk?



48. We are given $\sigma = 4.00 \times 10^{-6}$ C/m^2 and various values of z (in the notation of Eq. 22-26, which specifies the field E of the charged disk). Using this with $F = eE$ (the magnitude of Eq. 22-28 applied to the electron) and $F = ma$, we obtain $a = F/m = eE/m$.

(a) The magnitude of the acceleration at a distance R is

$$a = \frac{e\,\sigma\,(2-\sqrt{2})}{4\,m\,\varepsilon_0} = 1.16 \times 10^{16}\ \text{m/s}^2\ .$$

(b) At a distance R/100, $a = \dfrac{e\,\sigma\,(10001 - \sqrt{10001})}{20002\,m\,\varepsilon_0} = 3.94 \times 10^{16}\ \text{m/s}^2$.

(c) At a distance R/1000, $a = \dfrac{e\,\sigma\,(1000001 - \sqrt{1000001})}{2000002\,m\,\varepsilon_0} = 3.97 \times 10^{16}\ \text{m/s}^2$.

(d) The field due to the disk becomes more uniform as the electron nears the center point. One way to view this is to consider the forces exerted on the electron by the charges near the edge of the disk; the net force on the electron caused by those charges will decrease due to the fact that their contributions come closer to canceling out as the electron approaches the middle of the disk.

22.40. An electron with a speed of 5.00×10^8 cm/s enters an electric field of magnitude 1.00×10^3 N/C, traveling along a field line in the direction that retards its motion. (a) How far will the electron travel in the field before stopping momentarily, and (b) how much time will have elapsed? (c) If the region containing the electric field is 8.00 mm long (too short for the electron to stop within it), what fraction of the electron's initial kinetic energy will be lost in that region?

40. (a) The initial direction of motion is taken to be the $+x$ direction (this is also the direction of $\vec{E}$). We use $v_f^2 - v_i^2 = 2a\Delta x$ with $v_f = 0$ and $\vec{a} = \vec{F}/m = -e\vec{E}/m_e$ to solve for distance Δx:

$$\Delta x = \frac{-v_i^2}{2a} = \frac{-m_e v_i^2}{-2eE} = \frac{-(9.11\times10^{-31}\text{ kg})(5.00\times10^6\text{ m/s})^2}{-2(1.60\times10^{-19}\text{ C})(1.00\times10^3\text{ N/C})} = 7.12\times10^{-2}\text{ m}.$$

(b) Equation 2-17 leads to

$$t = \frac{\Delta x}{v_{\text{avg}}} = \frac{2\Delta x}{v_i} = \frac{2(7.12\times10^{-2}\text{ m})}{5.00\times10^6\text{ m/s}} = 2.85\times10^{-8}\text{ s}.$$

(c) Using $\Delta v^2 = 2a\Delta x$ with the new value of Δx, we find

$$\frac{\Delta K}{K_i} = \frac{\Delta\left(\frac{1}{2}m_e v^2\right)}{\frac{1}{2}m_e v_i^2} = \frac{\Delta v^2}{v_i^2} = \frac{2a\Delta x}{v_i^2} = \frac{-2eE\Delta x}{m_e v_i^2}$$
$$= \frac{-2(1.60\times10^{-19}\text{C})(1.00\times10^3\text{ N/C})(8.00\times10^{-3}\text{m})}{(9.11\times10^{-31}\text{kg})(5.00\times10^6\text{ m/s})^2} = -0.112.$$

Thus, the fraction of the initial kinetic energy lost in the region is 0.112 or 11.2%.

22.79. A clock face has negative point charges $-q, -2q, -3q, \ldots, -12q$ fixed at the positions of the corresponding numerals. The clock hands do not perturb the net field due to the point charges. At what time does the hour hand point in the same direction as the electric field vector at the center of the dial? (*Hint:* Use symmetry.)

79. We consider pairs of diametrically opposed charges. The net field due to just the charges in the one o'clock ($-q$) and seven o'clock ($-7q$) positions is clearly equivalent to that of a single $-6q$ charge sitting at the seven o'clock position. Similarly, the net field due to just the charges in the six o'clock ($-6q$) and twelve o'clock ($-12q$) positions is the same as that due to a single $-6q$ charge sitting at the twelve o'clock position. Continuing with this line of reasoning, we see that there are six equal-magnitude electric field vectors pointing at the seven o'clock, eight o'clock, … twelve o'clock positions. Thus, the resultant field of all these points, by symmetry, is directed toward the position midway between seven and twelve o'clock. Therefore, $\vec{E}_{\text{resultant}}$ points toward the nine-thirty position.