Visualizing Potential Surfaces with a Spreadsheet

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A common instructional approach to the topic of electric potential is to make comparisons to gravitational potential energy. Teachers often mention topographic maps and relate the lines of constant elevation (i.e., gravitational equipotentials) to their electrical counterparts. For example, a parallel plate capacitor is described as being like a ridge and valley that run parallel to each other. A bowling ball released between the two will roll "downhill" into the valley, mimicking the behavior of a positive charge released between the capacitor plates. This seems to help students grasp the main ideas, but can get cumbersome for more complex systems of charges. In any case, drawing the figures on the board is often an exercise in frustration.

The recognition of the instructional value of potential surface drawings is not new.[1] Computer programs that generate equipotential lines also have a long history.[2-4] The increasing capability of spreadsheets has resulted in their use for what previously would have required a custom-written program, including the production of contour plots.[5-7] For the most part, the relaxation technique of solving Laplace's equation has been the preferred method of calculation, whether implemented on a spreadsheet or in a higher-level language.[8-12] A considerably simpler approach was presented by Dykstra and Fuller.[13] They utilized the fact that the electric potential due to a point charge is given by

$$V = \frac{Q}{4\pi\varepsilon_0 r} \tag{1}$$

where Q is the quantity of charge establishing the potential and r is the radial distance from that charge to the location where one wants to know the potential. Dykstra and Fuller devised the clever scheme of using the columns and rows of the spreadsheet as a sort of rectangular coordinate system. By having a cell determine its row and column position relative
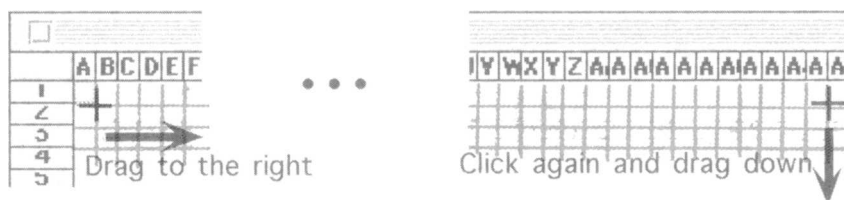

Fig. 1. Carefully placing the mouse in the lower right corner of A1 gives you a plus cursor. Drag the cursor to the right edge of your grid and release the mouse. This duplicates the formula into row 1 of the grid. Then place the mouse in the lower right corner of this row and drag the cursor down at the bottom right corner of your grid to fill each cell with the formula.

Fig. 2. Click on the naming field and type the name for the cell, in this case charge1. Press the Enter key to complete the cell-naming shortcut.

to another cell "holding" the charge, it becomes a simple matter to calculate V.

$$V \approx \frac{Q}{4\pi\varepsilon_0 \sqrt{(\text{difference in column numbers})^2 + (\text{difference in row numbers})^2}} \tag{2}$$

This technique is the key to developing three-dimensional surface plots. The x and y coordinates of any given point on the surface are simply the column and row numbers of a cell in the spreadsheet. The z coordinate is the potential calculated for that cell. By repeating the calculation in a large grid of small cells, a surface can easily be generated. And since potentials are scalars, it is a simple matter to incorporate additional charges and add their effects to the potential at each cell in the grid.
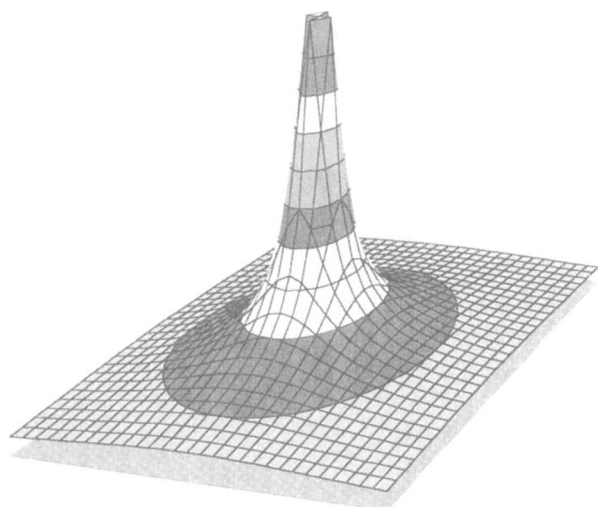
Fig. 3. Potential surface for a single positive charge. Scaling for the *x*- and *y*-axes is slightly different, resulting in oval cross sections rather than circular. Double clicking on the chart and then clicking on the edge of the gray enclosing "box" allows you to rotate the surface to change your point of view.
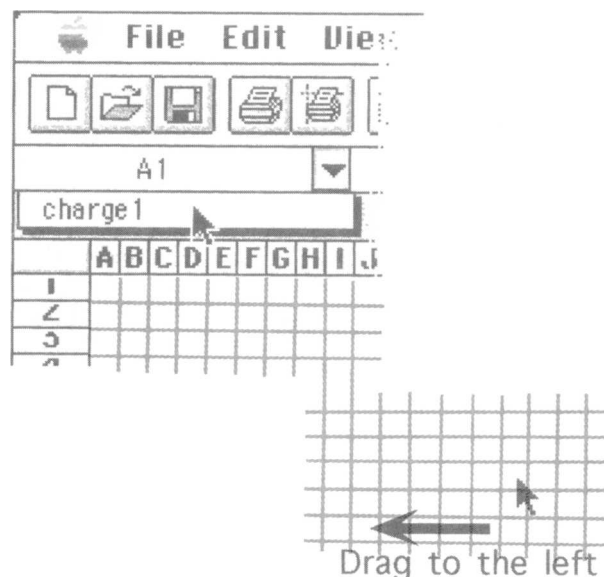

Fig. 4. Use the drop-down menu next to the naming field to select your charge1 cell. Then move the cursor over the cell until it turns into an arrow. Drag the cell toward the left approximately 10 columns.

Setting Up the Calculations

Although I will be describing the steps to take when using Microsoft Excel, other spreadsheet packages should have similar capabilities. Start with a new, empty spreadsheet.

1. Select all the cells and adjust their width and height until they are quite small and fairly square. (This allows you to work with a larger number of cells in your grid, resulting in a less "blocky" surface representation.)

2. In the top left cell, A1, enter the following formula:

= charge1/SQRT((COLUMN(A1) – COLUMN(charge1))^2 + (ROW(A1) – ROW(charge1))^2)

There are several things to notice about what you just entered. First, we have left out the constant $\frac{1}{4\pi\varepsilon_0}$ because it just scales the potential value. (If we had wanted to actually be able to read the calculated potential at a given cell, then it would be required. As it is, the spreadsheet will automatically scale the *z*-axis of our 3-D plot, so this simplification is justified.) Second, we are taking advantage of the naming capability of Excel. "charge1" will eventually be the name of the charge's location. Since we haven't established that location yet, Excel will not be able to carry out the calculation. You might see #NAME? appear in cell A1 where the software is trying to tell you that the charge1 doesn't exist.

3. Copy the formula into the rest of the cells in your grid. (Try to have at least 30 columns and 30 rows.) Either copy and paste or use the shortcut shown in Fig. 1.

4. Click on a cell somewhere near the center to hold your charge. I used R17. Type the number 10 into the cell. Any value will do for the charge, since Excel will automatically scale the *z*-axis of the graph. I tend to enter numbers between ±1 and ±50 and tell students to think of these as μC. In situations where you have more than a single charge in the grid only the relative sizes are important.

5. Now name the cell "charge1" (without the quote marks). The easiest way to do this in Excel is to click in the naming field to the left of the formula entry area (see Fig. 2) and just type in the name and press the Enter key. You may notice a change in your spreadsheet now that Excel can actually carry out the calculation.
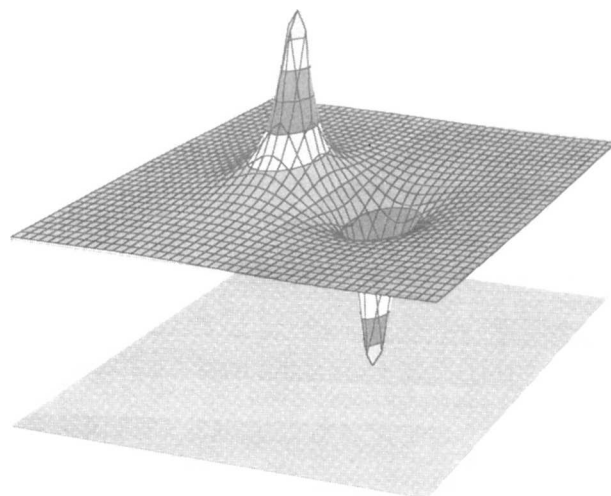

Fig. 5. Dipole potential surface. The number of cells in the grid was increased to increase accuracy near the charges.

6. Now drag from cell A1 to the opposite corner of your grid to select all the cells in it. Click on the "ChartWizard" button in the toolbar or select **Insert:Chart** from the menubar. A crosshair cursor will appear. Use it to drag a large rectangle in the Excel window, probably right over your grid unless you have a very large screen. This is where the chart will appear. This brings up a series of dialog boxes asking you to verify the grid area and specify the type of chart you would like to see. Step through these boxes, selecting a 3-D Surface graph that is filled in (not wire frame) and with a perspective view. A legend and a graph title are not necessary. Within a few seconds, the electric potential surface graph appears. Note that the speed of your computer will make a tremendous difference in how quickly the graph is created. Because of the automatic scaling of the axes, you may not get a symmetric potential surface. The equipotentials (which
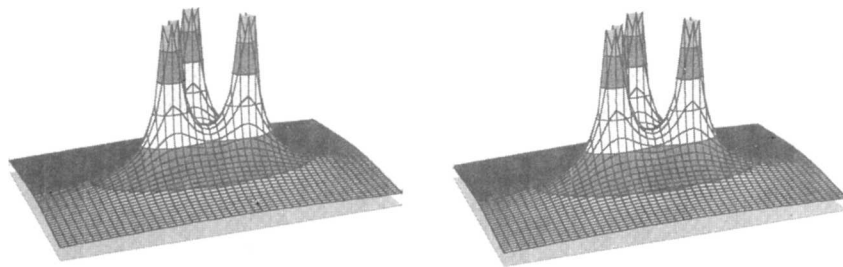
Fig. 6. Potential surfaces for three equal charges. Cross your eyes to create a stereoscopic effect.

are indicated by different colors on the chart) may appear as ovals as they do in Fig. 3, rather than circles.

This same procedure can be extended to any number of charges. For example, a dipole is modeled with the following formula in cell A1 (which can be made by a quick copy, paste, and revision of the original formula):

= charge1/SQRT((COLUMN(A1) – COLUMN(charge1))^2 + (ROW(A1) – ROW(charge1))^2) + charge2/SQRT((COLUMN(A1) – COLUMN(charge2))^2 + (ROW(A1) – ROW(charge2))^2)

You'll probably need to drag the chart out of the way before you copy the new formula into the rest of the grid. Don't be surprised if Excel tells you that it "Can't resolve circular references." As soon as you put the number 10 back into your charge1 cell, it will be fine. After you do this, drag the cell over to a new location, moving not only the contents but also the cell name. See Fig. 4.

Be sure to recopy the formula into the original location of the charge1 cell or you'll end up with a hole in your potential surface. Finally click on a location for the negative charge of the dipole. Enter –10 and name that cell "charge2," following the same procedure as before. The sheet will recalculate all the potentials for you, instantly updating the 3-D chart. If you moved it out of the way, drag it back so you can see the results of your handiwork. You should see something like Fig. 5.

By double clicking on the chart and then clicking on the edge of the grey enclosing box you can cause "handles" to appear that allow you to rotate the surface. It is often instructive to spin it so that you are looking straight down upon it, showing the equipotentials as they are usually drawn. If you are really adventuresome you can copy the entire chart and paste it back down beside the original. By making slight adjustments to the view and crossing your eyes you can get a very realistic stereoscopic effect. This is shown in Fig. 6 for a three-charge arrangement.

Using the Technique for Instruction

The steps are simple enough that students who are somewhat familiar with spreadsheet operation can design their own charge distributions and view potential surfaces from a variety of perspectives. I have found that projecting the computer screen during class while I work through an example is very helpful. This is a good time to show students the shortcuts for dragging to copy formulas, naming cells, going to a named cell by using the drop menu next to the name field, etc. Using the shortcuts can tremendously speed

up the process (which is why they are presented here in such detail).

Of course the real purpose is not to teach spreadsheet tricks, but to help students grasp the relationship between potential and the electric field. (See Gastineau et al.[14] for a tutorial whose purpose is to teach spreadsheets entirely outside of class.) The computer-aided visualization of the different situations seems to make it easier for them to recognize that $\vec{E}$ is just the negative gradient (i.e., the downhill slope) of the potential surface. In fact, I now teach the gradient as a mathematical tool, even in an introductory-level engineering physics course. Also, students appear to spontaneously understand the importance of an infinitesimally small test charge (so it doesn't distort the surface by making its own "dent" or "bump"). They also better understand the reason for setting the potential at infinity to zero. In their words, that's how far you'd have to go before the surface is really "flat." The three-charge scenario shown in Fig. 6 is helpful because students can easily see that the potential between the charges is not zero, even though part of the area is locally flat, thus having no electric field.

This technique appears to be quite popular with students. I have had positive responses from them and anecdotal evidence that it improves their understanding of the relationship between electric potential and the electric field. I find I can refer back to surfaces we had examined earlier without recreating them and students are able to mentally recall their shapes and imagine how charged particles would behave in response to the sloping potentials.

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