# Discussion Chapter 21: Coulomb's Law

**21.54** A charge of 6.0 $\mu$C is to be split into two parts that are then separated by 3.0 mm. What is the maximum possible magnitude of the electrostatic force between those two parts?

54. Let $q_1$ be the charge of one part and $q_2$ that of the other part; thus, $q_1 + q_2 = Q = 6.0\ \mu\text{C}$. The repulsive force between them is given by Coulomb's law:

$$F = \frac{q_1 q_2}{4\pi\varepsilon_0 r^2} = \frac{q_1(Q-q_1)}{4\pi\varepsilon_0 r^2} \ .$$

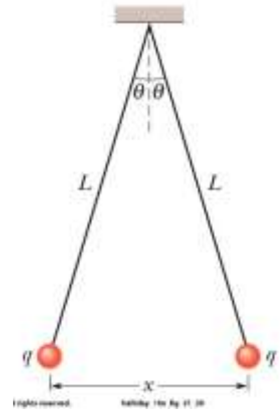
If we maximize this expression by taking the derivative with respect to $q_1$ and setting equal to zero, we find $q_1 = Q/2$ , which might have been anticipated (based on symmetry arguments). This implies $q_2 = Q/2$ also. With $r = 0.0030$ m and $Q = 6.0 \times 10^{-6}$ C, we find

$$F = \frac{(Q/2)(Q/2)}{4\pi\varepsilon_0 r^2} = \frac{1}{4}\frac{Q^2}{4\pi\varepsilon_0 r^2} = \frac{1}{4}\frac{\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(6.0\times10^{-6}\,\text{C}\right)^2}{\left(3.00\times10^{-3}\,\text{m}\right)^2} \approx 9.0\times10^3\ \text{N}\,.$$

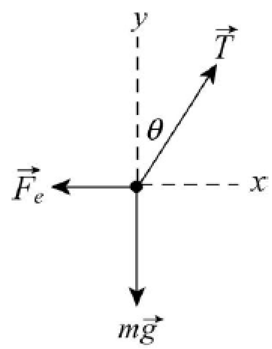
**21.42** In Fig. 21-39, two tiny conducting balls of identical mass $m$ and identical charge $q$ hang from nonconducting threads of length $L$. Assume that $\theta$ is so small that $\tan\theta$ can be replaced by its approximate equal, $\sin\theta$. (a) Show that

$$x = \left(\frac{q^2 L}{2\pi\varepsilon_0 mg}\right)^{1/3}$$

gives the equilibrium separation $x$ of the balls. (b) If $L$ = 120 cm, $m$ = 10 g, and $x$ = 5.0 cm, what is $|q|$?




42. (a) A force diagram for one of the balls is shown below. The force of gravity $m\vec{g}$ acts downward, the electrical force $\vec{F}_e$ of the other ball acts to the left, and the tension in the thread acts along the thread, at the angle $\theta$ to the vertical. The ball is in equilibrium, so its acceleration is zero. The $y$ component of Newton's second law yields $T\cos\theta - mg = 0$ and the $x$ component yields $T\sin\theta - F_e = 0$. We solve the first equation for $T$ and obtain $T = mg/\cos\theta$. We substitute the result into the second to obtain $mg\tan\theta - F_e = 0$.




Examination of the geometry of the figure shown leads to $\tan\theta = \dfrac{x/2}{\sqrt{L^2 - (x/2)^2}}$.

If $L$ is much larger than $x$ (which is the case if $\theta$ is very small), we may neglect $x/2$ in the denominator and write $\tan\theta \approx x/2L$. This is equivalent to approximating $\tan\theta$ by $\sin\theta$. The magnitude of the electrical force of one ball on the other is

$$F_e = \frac{q^2}{4\pi\varepsilon_0 x^2}$$

by Eq. 21-4. When these two expressions are used in the equation $mg\tan\theta = F_e$, we obtain

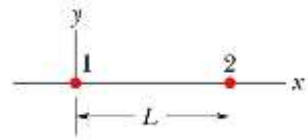
$$\frac{mgx}{2L} \approx \frac{1}{4\pi\varepsilon_0}\frac{q^2}{x^2} \quad\Rightarrow\quad x \approx \left(\frac{q^2 L}{2\pi\varepsilon_0 mg}\right)^{1/3}.$$

(b) We solve $x^3 = 2kq^2L/mg$ for the charge (using Eq. 21-5):

$$q = \sqrt{\frac{mgx^3}{2kL}} = \sqrt{\frac{(0.010\,\text{kg})(9.8\,\text{m/s}^2)(0.050\,\text{m})^3}{2(8.99\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2)(1.20\,\text{m})}} = \pm 2.4\times10^{-8}\,\text{C}.$$

Thus, the magnitude is $|q| = 2.4\times10^{-8}$ C.

**21.30** In Fig. 21-26, particles 1 and 2 are fixed in place on an $x$ axis, at a separation of $L = 8.00$ cm. Their charges are $q_1 = +e$ and $q_2 = -27e$. Particle 3 with charge $q_3 = +4e$ is to be placed on the line between particles 1 and 2, so that they produce a net electrostatic force $\vec{\mathbf{F}}_{3,net}$ on it. (a) At what coordinate should particle 3 be placed to minimize the magnitude of that force? (b) What is that minimum magnitude?




30. (a) Let $x$ be the distance between particle 1 and particle 3. Thus, the distance between particle 3 and particle 2 is $L - x$. Both particles exert leftward forces on $q_3$ (so long as it is on the line between them), so the magnitude of the net force on $q_3$ is

$$F_{net} = |\vec{F}_{13}| + |\vec{F}_{23}| = \frac{|q_1 q_3|}{4\pi\varepsilon_0 x^2} + \frac{|q_2 q_3|}{4\pi\varepsilon_0 (L-x)^2} = \frac{e^2}{\pi\varepsilon_0}\left(\frac{1}{x^2} + \frac{27}{(L-x)^2}\right)$$

with the values of the charges (stated in the problem) plugged in. Finding the value of $x$ that minimizes this expression leads to $x = \frac{1}{4} L$. Thus, $x = 2.00$ cm.*

(b) Substituting $x = \frac{1}{4} L$ back into the expression for the net force magnitude and using the standard value for $e$ leads to $F_{net} = 9.21 \times 10^{-24}$ N.

*Here's how to find the value of $x$ that minimizes the net force. First take the derivative with respect to $x$ and set it equal to zero:

$$\frac{d}{dx}\left[\frac{e^2}{\pi\varepsilon_0}\left(\frac{1}{x^2} + \frac{27}{(L-x)^2}\right)\right] = \frac{e^2}{\pi\varepsilon_0}\left[-2x^{-3} + (-2)27(L-x)^{-3}(-1)\right] \overset{\text{set}}{=} 0$$

$$\frac{54}{(L-x)^3} = \frac{2}{x^3}$$

$$27x^3 = (L-x)^3$$

$$\sqrt[3]{27}\ x = L - x$$

$$4x = L$$

# Discussion Chapter 22: Electric Fields

**22.04**. Two charged particles are attached to an $x$ axis: Particle 1 of charge $-2.00 \times 10^{-7}$ C is at position $x = 6.00$ cm and particle 2 of charge $+2.00 \times 10^{-7}$ C is at position $x = 21.0$ cm. Midway between the particles, what is their net electric field in unit-vector notation?

4. With $x_1 = 6.00$ cm and $x_2 = 21.00$ cm, the point midway between the two charges is located at $x = 13.5$ cm. The values of the charge are

$$q_1 = -q_2 = -2.00 \times 10^{-7}\ \text{C},$$

and the magnitudes and directions of the individual fields are given by:

$$\vec{E}_1 = -\frac{|q_1|}{4\pi\varepsilon_0 (x - x_1)^2}\hat{\text{i}} = -\frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)|-2.00\times10^{-7}\ \text{C}|}{(0.135\ \text{m} - 0.060\ \text{m})^2}\hat{\text{i}} = -(3.196\times10^5\ \text{N/C})\hat{\text{i}}$$

$$\vec{E}_2 = -\frac{q_2}{4\pi\varepsilon_0 (x - x_2)^2}\hat{\text{i}} = -\frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(2.00\times10^{-7}\ \text{C})}{(0.135\ \text{m} - 0.210\ \text{m})^2}\hat{\text{i}} = -(3.196\times10^5\ \text{N/C})\hat{\text{i}}$$

Thus, the net electric field is $\vec{E}_{\text{net}} = \vec{E}_1 + \vec{E}_2 = -(6.39\times10^5\ \text{N/C})\hat{\text{i}}$.