Discussion Examples
Chapter 13: Oscillations About Equilibrium

17. The position of a mass on a spring is given by \( x = (6.5 \text{ cm}) \cos\left(\frac{2\pi t}{0.88 \text{ s}}\right) \). (a) What is the period, \( T \), of this motion? (b) Where is the mass at \( t = 0.25 \text{ s} \)? (c) Show that the mass is at the same location at \( 0.25 \text{ s} + T \) seconds as it is at \( 0.25 \text{ s} \).

**Picture the Problem:** A mass is attached to a spring. The mass is displaced from equilibrium and released from rest. The spring force causes the mass to oscillate about the equilibrium position in harmonic motion.

**Strategy:** The period can be obtained directly from the argument of the cosine function. Substituting the specific time into the equation will yield the location at that time. Substituting in the new time will show that the mass is at the same location one period later.

**Solution:**

1. (a) Identify the period \( T \) from the cosine equation: \( x = A \cos\left(\frac{2\pi t}{T}\right) \), so here \( T = 0.88 \text{ s} \).

2. (b) Substitute \( t = 0.25 \text{ s} \) into the equation and evaluate \( x \):

\[
x = (6.5 \text{ cm}) \cos\left(\frac{2\pi (0.25 \text{ s})}{0.88 \text{ s}}\right) = -1.4 \text{ cm}
\]

3. (c) Substitute \( t = (0.25 \text{ s} + T) \) into the equation and factor:

\[
x = A \cos\left(\frac{2\pi}{T}(0.25 \text{ s} + T)\right) = A \cos\left(\frac{2\pi}{T}(0.25 \text{ s} + 2\pi)\right)
\]

4. Drop the \( 2\pi \) phase shift, because \( \cos(x + 2\pi) = \cos(x) \):

\[
x = A \cos\left(\frac{2\pi}{T}0.25 \text{ s}\right)
\]

5. Insert the numeric values:

\[
x = (6.5 \text{ cm}) \cos\left(\frac{2\pi (0.25 \text{ s})}{0.88 \text{ s}}\right) = -1.4 \text{ cm} \Rightarrow \text{same location}
\]

**Insight:** Increasing the time by any multiple of the period increases the argument of the cosine function by the same multiple of \( 2\pi \), which has no effect upon the value of the cosine function.

44. **IP** The springs of a 511-kg motorcycle have an effective force constant of 9130 N/m. (a) If a person sits on the motorcycle, does its period of oscillation increase, decrease, or stay the same? (b) By what percent and in what direction does the period of oscillation change when a 112-kg person rides the motorcycle?

**Picture the Problem:** If the motorcycle is pushed down slightly on its springs it will oscillate up and down in harmonic motion. A rider sitting on the motorcycle effectively increases the mass of the motorcycle and oscillates also.

**Strategy:** We can use the equation for the period of a mass on a spring. Writing this equation for the motorcycle without rider and again for the motorcycle with rider we can calculate the percent difference in the periods.

**Solution:**

1. (a) The period increases because the person’s mass is added to the system and \( T \propto \sqrt{m} \).

2. (b) Write the equation for the period of the motorcycle without the rider:

\[
T = 2\pi \sqrt{\frac{m}{k}}
\]

3. Write the equation for the period of the motorcycle with the rider:

\[
T_2 = 2\pi \sqrt{\frac{m+M}{k}}
\]

4. Calculate the percent difference between the two periods:

\[
\frac{T_2 - T}{T} = \frac{2\pi \sqrt{\frac{m+M}{k}} - 2\pi \sqrt{\frac{m}{k}}}{2\pi \sqrt{\frac{m}{k}}}
\]
5. Simplify by factoring out $2\pi \sqrt{m/k}$ from the numerator and denominator:

$$\frac{T_2 - T_1}{T} = \frac{\sqrt{\frac{m+M}{m}} - 1}{\sqrt{\frac{511+122}{511}} - 1} = \frac{0.104}{511 + 122 - 511} = 0.104 = \boxed{0.4\%}$$

**Insight:** The percent change in the period does not depend on the spring force constant. It only depends on the fractional increase in mass.

50. **IP** A 0.40-kg mass is attached to a spring with a force constant of 26 N/m and released from rest a distance of 3.2 cm from the equilibrium position of the spring. (a) Give a strategy that allows you to find the speed of the mass when it is halfway to the equilibrium position. (b) Use your strategy to find this speed.

**Picture the Problem:** A mass attached to a spring is stretched from equilibrium position.

**Strategy:** The work done in stretching the spring is stored as potential energy in the spring until the mass is released. After the mass is released, the mass will accelerate, converting the potential energy into kinetic energy. The energy will then transfer back and forth between potential and kinetic energies as the mass oscillates about the equilibrium position.

**Solve the conservation of mechanical energy equation,** $E = K + U$, for the kinetic energy.

$$\frac{1}{2}mv^2 = \frac{1}{2}kA^2 - \frac{1}{2}k\left(\frac{1}{2}A\right)^2$$

**Solution:**

1. Set $K = E - U$ and substitute expressions for each term:

$$\frac{1}{2}mv^2 = \frac{1}{2}kA^2 - \frac{1}{2}k\left(\frac{1}{2}A\right)^2$$

2. Solve for the speed and simplify:

$$v = \sqrt{\frac{k\left(A^2 - \left(\frac{1}{2}A\right)^2\right)}{m}} = \sqrt{\frac{3kA^2}{4m}}$$

3. Insert the numeric values:

$$v = \sqrt{\frac{3(26 \text{ N/m})(0.032 \text{ m})^2}{4(0.40 \text{ kg})}} = 0.22 \text{ m/s}$$

**Insight:** When the displacement is half the maximum displacement, the speed is not half the maximum speed. In fact, the speed is $\frac{\sqrt{3}}{2}v_{\text{max}}$, which is greater than half the speed.

61. **United Nations Pendulum** A large pendulum with a 200-lb gold-plated bob 12 inches in diameter is on display in the lobby of the United Nations building. The pendulum has a length of 75 ft. It is used to show the rotation of the Earth—for this reason it is referred to as a Foucault pendulum. What is the least amount of time it takes for the bob to swing from a position of maximum displacement to the equilibrium position of the pendulum? (Assume that the acceleration due to gravity is $g = 9.81 \text{ m/s}^2$ at the UN building.)

**Picture the Problem:** The pendulum mass is displaced slightly from equilibrium and oscillates back and forth through the vertical.

**Strategy:** The time the pendulum takes to move from maximum displacement to equilibrium position is one-quarter of a period. Use equation 13-20 to determine the time.

**Solution:** Insert the numeric values into equation 13-20 and convert feet to meters:

$$\frac{T}{4} = \frac{\pi}{2} \sqrt{\frac{L}{g}} = \frac{\pi}{2} \sqrt{\frac{75.0 \text{ ft}}{9.81 \text{ m/s}^2 \left(\frac{0.305 \text{ m}}{\text{ft}}\right)}} = 2.4 \text{ s}$$

**Insight:** The full period of this pendulum is $4(2.4 \text{ s}) = 9.6 \text{ seconds}$. A pendulum with only half this length would have a period of 6.8 s.
14. A brother and sister try to communicate with a string tied between two tin cans (Figure 14–33). If the string is 9.5 m long, has a mass of 32 g, and is pulled taut with a tension of 8.6 N, how much time does it take for a wave to travel from one end of the string to the other?

**Picture the Problem:** The image shows two people talking on a tin can telephone. The cans are connected by a 9.5-meter-long string weighing 32 grams.

**Strategy:** Set the time equal to the distance divided by the velocity, where the velocity is given by equation 14-2. The linear mass density is the total mass divided by the length.

**Solution:** 1. Set the time equal to the distance divided by velocity:

\[ t = \frac{d}{v} = \frac{d}{\sqrt{\frac{\mu}{F}}} \]

2. Substitute \( \mu = \frac{m}{d} \) and insert numerical values:

\[ t = d \sqrt{\frac{m}{d}} \sqrt{\frac{d}{F}} = \sqrt{\frac{(0.032 \text{ kg})(9.5 \text{ m})}{8.6 \text{ N}}} = 0.19 \text{ s} \]

**Insight:** The message travels the same distance in the air in 0.028 seconds, about 7 times faster.

38. In a pig-calling contest, a caller produces a sound with an intensity level of 110 dB. How many such callers would be required to reach the pain level of 120 dB?

**Picture the Problem:** We are given the sound intensity of one pig caller and are asked to calculate how many pig callers are needed to increase the intensity level by 10 dB.

**Strategy:** Multiply the intensity in equation 14-8 by \( N \) callers, setting the intensity level to 120 dB and solve for \( N \).

**Solution:** 1. Write the intensity level for \( N \) callers:

\[ \beta = 10 \log \left( \frac{NI}{I_0} \right) = 10 \log (N) + 10 \log \left( \frac{I}{I_0} \right) \]

2. Insert the intensity levels and solve for \( N \):

\[ 120 \text{ dB} = 10 \log (N) + 110 \text{ dB} \]
\[ 10 \text{ dB} = 10 \log (N) \]

\[ N = 10^{10/10} = 10 \text{ callers} \]

**Insight:** Increasing the intensity level by 10 dB increases the intensity by a factor of 10. Therefore 10 callers, each with intensity level 110 dB, would produce a net intensity level of 120 dB. 100 callers (10 \times 10 callers) would be needed to produce an intensity level of 130 dB (120 dB + 10 dB).
45. A train moving with a speed of 31.8 m/s sounds a 136-Hz horn. What frequency is heard by an observer standing near the tracks as the train approaches?

**Picture the Problem:** The train, a moving source, sounds its horn. We wish to calculate the frequency heard by a person standing near the tracks.

**Strategy:** Solve equation 14-10 for the observed frequency, using the negative sign because the train is moving toward the observer.

**Solution:** Insert the given data into equation 14-10:

\[
 f' = \left( \frac{1}{1-u/v} \right) f = \frac{1}{1-\left( \frac{31.8 \text{ m/s}}{343 \text{ m/s}} \right)} \quad (136 \text{ Hz})
\]

\[
 f' = \frac{1}{1-\left( \frac{31.8 \text{ m/s}}{343 \text{ m/s}} \right)} \quad (136 \text{ Hz}) = 1.50 \times 10^2 \text{ Hz}
\]

**Insight:** If the train were moving away from the observer, he would hear a frequency of 124 Hz.

62. **IP** Two violinists, one directly behind the other, play for a listener directly in front of them. Both violinists sound concert A (440 Hz). (a) What is the smallest separation between the violinists that will produce destructive interference for the listener? (b) Does this smallest separation increase or decrease if the violinists produce a note with a higher frequency? (c) Repeat part (a) for violinists who produce sounds of 540 Hz.

**Picture the Problem:** Two violinists separated by a distance \( d \), as shown in the figure, play a 440-Hz note.

**Strategy:** We want to calculate the smallest distance \( d \), for which the listener will hear destructive interference. Assume that the violins are in phase with each other. The smallest separation that will produce destructive interference occurs when the separation is equal to one-half of a wavelength. Set the distance to half a wavelength and use equation 14-1 to write the wavelength in terms of the frequency and speed of sound.

**Solution:** 1. (a) Set the distance equal to half a wavelength:

\[
 d = \frac{\lambda}{2} = \frac{v}{2f} = \frac{343 \text{ m/s}}{2 \times 440 \text{ Hz}} = 0.390 \text{ m}
\]

2. (b) The frequency is inversely proportional to the separation distance. Therefore, higher frequency means shorter minimum separation.

3. (c) Solve for the distance at 540 Hz:

\[
 d = \frac{\lambda}{2} = \frac{343 \text{ m/s}}{2 \times 540 \text{ Hz}} = 0.318 \text{ m}
\]

**Insight:** In order for the destructive interference to occur, the violins’ notes must be coherent and in phase with each other. This typically does not occur during a concert.
71. **IP BIO Standing Waves in the Human Ear** The human ear canal is much like an organ pipe that is closed at one end (at the tympanic membrane or eardrum) and open at the other (see the figure below). A typical ear canal has a length of about 2.4 cm. (a) What is the fundamental frequency and wavelength of the ear canal? (b) Find the frequency and wavelength of the ear canal’s third harmonic. (Recall that the third harmonic in this case is the standing wave with the second-lowest frequency.) (c) Suppose a person has an ear canal that is shorter than 2.4 cm. Is the fundamental frequency of that person’s ear canal greater than, less than, or the same as the value found in part (a)? Explain. (Note that the frequencies found in parts (a) and (b) correspond closely to the frequencies of enhanced sensitivity in Figure 14–28.)

**Picture the Problem:** The image shows an ear canal of length 2.4 cm.

**Strategy:** Treating the ear canal as a pipe closed at one end, we wish to calculate the fundamental and third harmonic frequencies and wavelengths. Use equation 14–14 to calculate the frequencies and wavelengths. For the fundamental use \( n = 1 \), and for the third harmonic use \( n = 3 \).

**Solution:**

1. (a) Calculate the fundamental frequency and wavelength using equation 14–14 with \( n = 1 \):

\[
\frac{f_1}{4L} = \frac{1(343 \text{ m/s})}{4(2.4 \times 10^{-2} \text{ m})} = 3.6 \text{ kHz}
\]

\[
\lambda_1 = 4L/n = 4(2.4 \text{ cm})/1 = 9.6 \text{ cm}
\]

2. (b) Calculate the third harmonic frequency and wavelength using equation 14–14 with \( n = 3 \):

\[
\frac{f_3}{4L} = \frac{3(343 \text{ m/s})}{4(2.4 \times 10^{-2} \text{ m})} = 11 \text{ kHz}
\]

\[
\lambda_3 = 4L/n = 4(2.4 \text{ cm})/3 = 3.2 \text{ cm}
\]

3. (c) The fundamental frequency is inversely proportional to the length of the ear canal. Therefore, if an ear canal is shorter than 2.4 cm, the fundamental frequency of that person’s ear canal is **greater than** the value found in part (a).

**Insight:** For an ear canal of length 2.2 cm the fundamental frequency will be 3.9 kHz.

73. **IP** A 12.5-g clothesline is stretched with a tension of 22.1 N between two poles 7.66 m apart. What is the frequency of (a) the fundamental and (b) the second harmonic? (c) If the tension in the clothesline is increased, do the frequencies in parts (a) and (b) increase, decrease, or stay the same? Explain.

**Picture the Problem:** The image shows two clotheslines that are 7.66 m long. One line is oscillating at the fundamental frequency and the other at the second harmonic.

**Strategy:** First use the tension and mass to calculate the speed of the waves, using equation 14–2. Then use equation 14–13 to calculate the frequencies.

**Solution:**

1. (a) Solve equation 14–2 for the wave speed:

\[
v = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{22.1 \text{ N}}{0.0125 \text{ kg} / 7.66 \text{ m}}} = 116.4 \text{ m/s}
\]

2. Set \( n = 1 \) in equation 14–13 to calculate the fundamental frequency:

\[
f_1 = \frac{nv}{2L} = \frac{1(116.4 \text{ m/s})}{2(7.66 \text{ m})} = 7.60 \text{ Hz}
\]

3. (b) Set \( n = 2 \) in equation 14–13 to calculate the second harmonic frequency:

\[
f_2 = \frac{nv}{2L} = \frac{2(116.4 \text{ m/s})}{2(7.66 \text{ m})} = 15.2 \text{ Hz}
\]

4. (c) The wave speed is proportional to the square root of the tension, and the frequency is proportional to the wave speed. We conclude that if the tension in the clothesline is increased, the frequencies in parts (a) and (b) will **increase**.

**Insight:** If the tension were doubled to 44.2 N, the frequencies would increase by a factor of \( \sqrt{2} \) to \( f_1 = 10.7 \text{ Hz} \) and \( f_2 = 21.5 \text{ Hz} \). We bent the rules for significant figures a little in step 1 in order to avoid rounding error.