Discussion Examples
Chapter 10: Rotational Kinematics and Energy

9. **The Crab Nebula** One of the most studied objects in the night sky is the Crab nebula, the remains of a supernova explosion observed by the Chinese in 1054. In 1968 it was discovered that a pulsar—a rapidly rotating neutron star that emits a pulse of radio waves with each revolution—lies near the center of the Crab nebula. The period of this pulsar is 33 ms. What is the angular speed (in rad/s) of the Crab nebula pulsar?

Picture the Problem: The pulsar rotates about its axis, completing 1 revolution in 0.33 s.

Strategy: Divide one revolution or 2π radians by the period in seconds to find the angular speed.

Solution: Calculate ω using equation 10-3:

$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{2\pi \text{ rad}}{T} = \frac{2\pi \text{ rad}}{0.033\text{s}} = \boxed{190 \text{ rad/s}}$$

Insight: The rotation rate of the pulsar can also be described as 1800 rev/min.

20. A discus thrower starts from rest and begins to rotate with a constant angular acceleration of 2.2 rad/s^2. **(a)** How many revolutions does it take for the discus thrower's angular speed to reach 6.3 rad/s? **(b)** How much time does this take?

Picture the Problem: The discus thrower rotates about a vertical axis through her center of mass, increasing her angular velocity at a constant rate.

Strategy: Use the kinematic equations for rotation (equations 10-8 through 10-11) to find the number of revolutions through which the athlete rotates and the time elapsed during the specified interval.

Solution: 1. (a) Solve equation 10-11 for $\Delta\theta$:

$$\Delta\theta = \theta - \theta_0 = \frac{\omega^2 - \omega_0^2}{2\alpha} = \frac{(6.3 \text{ rad/s})^2 - 0^2}{2\left(2.2 \text{ rad/s}^2\right)} = 9.0 \text{ rad} \times 1 \text{ rev}/2\pi \text{ rad}$$

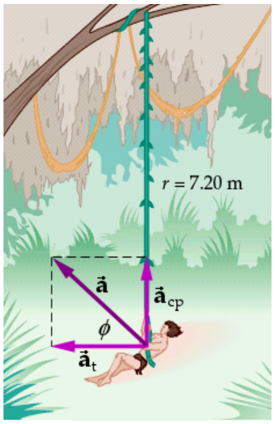
$$= \boxed{1.4 \text{ rev}}$$

2. (b) Solve equation 10-8 for *t*:

$$t = \frac{\omega - \omega_0}{\alpha} = \frac{6.3 - 0 \text{ rad/s}}{2.2 \text{ rad/s}^2} = \boxed{2.9 \text{ s}}$$

Insight: Notice the athlete turns nearly one and a half times around. Therefore, she should begin her spin with her back turned toward the range if she plans to throw the discus after reaching 6.3 rad/s. If she does let go at that point, the linear speed of the discus will be about 6.3 m/s (for a 1.0 m long arm) and will travel about 4.0 m if launched at 45° above level ground. Not that great compared with a championship throw of over 40 m (130 ft) for a college woman.

35. **IP** Jeff of the Jungle swings on a vine that is 7.20 m long. Suppose that at some point in his swing Jeff has an angular speed of 0.850 rad/s and an angular acceleration of 0.620 rad/s^2. Find the magnitude of his centripetal, tangential, and total accelerations, and the angle his total acceleration makes with respect to the tangential direction of motion.



Picture the Problem: Jeff clings to a vine and swings along a vertical arc.

Strategy: Use equation 10-13 to find Jeff's centripetal acceleration and equation 10-14 to find his tangential acceleration. Add these two perpendicular vectors to find the total acceleration.

Solution: 1. Apply equation 10-13 directly:

$$a_{cp} = r\omega^2 = (7.20 \text{ m})(0.850 \text{ rad/s})^2 = \boxed{5.20 \text{ m/s}^2}$$

2. Apply equation 10-14 directly:

$$a_t = r\alpha = (7.20 \text{ m})\left(0.620 \text{ rad/s}^2\right) = \boxed{4.46 \text{ m/s}^2}$$

3. Add the two perpendicular vectors:

$$a = \sqrt{a_{cp}^2 + a_t^2} = \sqrt{\left(5.202 \text{ m/s}^2\right)^2 + \left(4.464 \text{ m/s}^2\right)^2} = \boxed{6.85 \text{ m/s}^2}$$

4. Find the angle ϕ:

$$\phi = \tan^{-1}\left(\frac{a_{cp}}{a_t}\right) = \tan^{-1}\left(\frac{5.20 \text{ m/s}^2}{4.46 \text{ m/s}^2}\right) = \boxed{49.4°}$$

Insight: The angle ϕ will increase with Jeff's speed if his angular acceleration remains constant because a_{cp} depends on the square of the tangential speed.

50. As you drive down the road at 17 m/s, you press on the gas pedal and speed up with a uniform acceleration of 1.12 m/s^2 for 0.65 s. If the tires on your car have a radius of 33 cm, what is their angular displacement during this period of acceleration?

Picture the Problem: Your car's tires roll without slipping, increasing their velocity at a constant rate.

Strategy: Use the fact that the tires roll without slipping to find the angular acceleration and angular velocity from their linear counterparts. Then use the kinematic equations for rotation (equations 10-8 through 10-11) to determine the angle through which the tire rotated during the specified interval.

Solution: 1. Solve equation 10-12 for ω_0:

$$\omega_0 = \frac{v_0}{r} = \frac{17\text{ m/s}}{0.33\text{ m}} = \underline{\underline{52\text{ rad/s}}}$$

2. Solve equation 10-14 for α:

$$\alpha = \frac{a}{r} = \frac{1.12\text{ m/s}^2}{0.33\text{ m}} = \underline{\underline{3.4\text{ rad/s}^2}}$$

3. Apply equation 10-10 directly:

$$\theta = \omega_0 t + \tfrac{1}{2}\alpha t^2 = (52\text{ rad/s})(0.65\text{ s}) + \tfrac{1}{2}(3.4\text{ rad/s}^2)(0.65\text{ s})^2$$
$$= \boxed{35\text{ rad}} = 5.5\text{ rev}$$

Insight: Another way to solve this question is to find the final angular speed (54 rad/s) and then use $\theta = \tfrac{1}{2}(\omega + \omega_0)t$ to find the answer.

This question is MUCH more difficult than you are expected to handle in this course, but it is very interesting so I include it here:

63. Find the rate at which the rotational kinetic energy of the Earth is decreasing. The Earth has a moment of inertia of $0.331 M_E R_E{}^2$, where $R_E = 6.38\times10^6\text{ m}$ and $M_E = 5.97\times10^{24}\text{ kg}$, and its rotational period increases by 2.3 ms with each passing century. Give your answer in watts.

Picture the Problem: The Earth rotates on its axis, slowing down with constant angular acceleration.

Strategy: Determine the difference in rotation rates over the span of a century by approximating $T + \Delta T \cong T$ because 0.0023 s is tiny compared with the time (86,400 s) it takes to complete one revolution. Then use equation 10-6 to find the average angular acceleration over the 100-year time interval.

Solution: 1. Find the difference in angular speeds:

$$\omega - \omega_0 = \frac{\theta}{T+\Delta T} - \frac{\theta}{T} = \theta\left[\frac{T-(T+\Delta T)}{T(T+\Delta T)}\right] \cong \theta\left(\frac{-\Delta T}{T^2}\right)$$
$$= (365\text{ rev}\times 2\pi\text{ rad/rev})\left\{\frac{-(0.840\text{ s})}{\left[(365\text{ d})(24\text{ h/d})(3600\text{ s/h})\right]^2}\right\}$$
$$\omega - \omega_0 = \underline{\underline{-1.94\times10^{-12}\text{ rad/s}}}$$

2. Find the sum of the angular speeds:

$$\omega + \omega_0 = \frac{\theta}{T+\Delta T} + \frac{\theta}{T} = \theta\left[\frac{T+(T+\Delta T)}{T(T+\Delta T)}\right] \cong \theta\left(\frac{2T+\Delta T}{T^2}\right) \cong \frac{2\theta}{T}$$

3. Multiply the results of steps 1 and 2:

$$(\omega-\omega_0)(\omega+\omega_0) = \omega^2 - \omega_0^2 = \left(-\theta\frac{\Delta T}{T^2}\right)\left(\frac{2\theta}{T}\right) = -\frac{2\theta^2\Delta T}{T^3}$$

4. Find the difference ΔK_r over a time interval of 100 years:

$$\Delta K_r = \tfrac{1}{2}I\omega^2 - \tfrac{1}{2}I\omega_0^2 = \tfrac{1}{2}I(\omega^2-\omega_0^2) = \tfrac{1}{2}I\left(-\frac{2\theta^2\Delta T}{T^3}\right) = -\frac{I\theta^2\Delta T}{T^3}$$
$$= -\frac{0.331 M_E R_E^2\theta^2\Delta T}{T^3}$$
$$= -\frac{0.331(5.97\times10^{24}\text{ kg})(6.38\times10^6\text{ m})^2(2\pi)^2(0.0023\text{ s})}{(86{,}400\text{ s})^2} = \underline{\underline{-1.1\times10^{22}\text{ J}}}$$

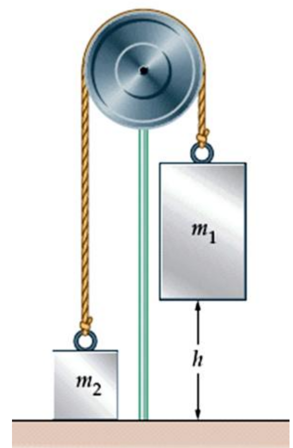
5. Use equation 7-10 to find the energy loss rate (power):

$$P = \frac{W}{t} = \frac{-1.1\times10^{22}\text{ J}}{100\text{ y}\times 3.16\times10^{7}\text{ s/y}} = \boxed{-3.5\times10^{12}\text{ W}} = -3.5\text{ TW}$$

Insight: Your first instinct might be to find the angular speed a hundred years ago assuming a period of 24.000 hrs (7.272205217×10^{-5} rad/s) and figure out the angular speed in 2010 (7.272205023×10^{-5} rad/s), but as you can see, attempting to subtract these numbers requires us to ignore the rules for significant figures. Using the approximation outlined above allows us to avoid the subtraction problem and keep two significant figures. The huge energy loss is due primarily to tidal friction, as the ocean tides dissipate the kinetic energy of the Earth's rotation into heat.

70. **IP Atwood's Machine** The two masses ($m_1 = 5.0$ kg and $m_2 = 3.0$ kg) in the Atwood's machine shown below are released from rest, with m_1 at a height of 0.75 m above the floor. When m_1 hits the ground its speed is 1.8 m/s. Assuming that the pulley is a uniform disk with a radius of 12 cm, **(a)** outline a strategy that allows you to find the mass of the pulley. **(b)** Implement the strategy given in part (a) and determine the pulley's mass.

Picture the Problem: The larger mass falls and the smaller mass rises until the larger mass hits the floor.



Strategy: Use conservation of mechanical energy, including the rotational energy of the pulley, to determine the mass of the pulley. Because the rope does not slip on the pulley, there is a direct relationship $v = r_p\omega$ between the rotation of the pulley and the linear speed of the rope and masses.

Solution: 1. (a) Equate the initial and final mechanical energies, then solve for the mass of the pulley.

2. (b) Set $E_i = E_f$ and let $\omega = v/r_p$:

$$U_i + K_i = U_f + K_f$$
$$m_1gh + 0 + 0 = m_2gh + \tfrac{1}{2}m_1v^2 + \tfrac{1}{2}m_2v^2 + \tfrac{1}{2}I_p\omega^2$$
$$(m_1 - m_2)gh = \tfrac{1}{2}(m_1 + m_2)v^2 + \tfrac{1}{2}\left(\tfrac{1}{2}m_pr_p^{\,2}\right)(v/r_p)^2$$

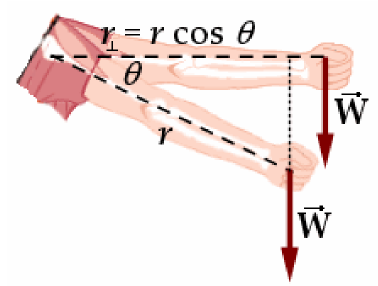
3. Rearrange the equation and solve for m_p :

$$\tfrac{1}{4}m_pv^2 = (m_1 - m_2)gh - \tfrac{1}{2}(m_1 + m_2)v^2$$
$$m_p = \frac{4\left[(m_1 - m_2)gh - \tfrac{1}{2}(m_1 + m_2)v^2\right]}{v^2}$$
$$= \frac{4\left[(2.0\text{ kg})(9.81\text{ m/s}^2)(0.75\text{ m}) - \tfrac{1}{2}(8.0\text{ kg})(1.8\text{ m/s})^2\right]}{(1.8\text{ m/s})^2}$$
$$m_p = \boxed{2.2\text{ kg}}$$

Insight: By the time the masses reach 1.8 m/s, 1.8 J or 12% of the 15 J of total kinetic energy is stored in the kinetic energy of the pulley, so the pulley plays a minor role in the energy balance of the system.

Chapter 11: Rotational Dynamics and Static Equilibrium

3. A 1.61 kg bowling trophy is held at arm's length, a distance of 0.605 m from the shoulder joint. What torque does the trophy exert about the shoulder if the arm is **(a)** horizontal, or **(b)** at an angle of 22.5° below the horizontal?



Picture the Problem: The arm extends out either horizontally or at some angle below horizontal, and the weight of the trophy is exerted straight downward on the hand.

Strategy: The torque equals the moment arm times the force according to equation 11-3. In this case the moment arm is the horizontal distance between the shoulder and the hand, and the force is the downward weight of the trophy. Find the horizontal distance in each case and multiply it by the weight of the trophy to find the torque. In part (b) the horizontal distance is $r_\perp = r\cos\theta = (0.605\text{ m})\cos 22.5° = 0.559\text{ m}.$

Solution: 1. (a) Multiply the moment arm by the weight: $\tau = r_\perp mg = (0.605\text{ m})(1.61\text{ kg})(9.81\text{ m/s}^2) = \boxed{9.56\text{ N}\cdot\text{m}}$

2. (b) Multiply the moment arm by the weight: $\tau = r_\perp mg = (0.559\text{ m})(1.61\text{ kg})(9.81\text{ m/s}^2) = \boxed{8.83\text{ N}\cdot\text{m}}$

Insight: The torque on the arm is reduced as the arm is lowered. The torque is exactly zero when the arm is vertical.

14. **IP** A wheel on a game show is given an initial angular speed of 1.22 rad/s. It comes to rest after rotating through 0.75 of a turn. **(a)** Find the average torque exerted on the wheel given that it is a disk of radius 0.71 m and mass 6.4 kg. **(b)** If the mass of the wheel is doubled and its radius is halved, will the angle through which it rotates before coming to rest increase, decrease, or stay the same? Explain. (Assume that the average torque exerted on the wheel is unchanged.)

Picture the Problem: The wheel rotates about its axis, decreasing its angular speed at a constant rate, and comes to rest.

Strategy: Use Table 10-1 to find the moment of inertia of a uniform disk and calculate I. Then use equation 10-11 to find the angular acceleration from the initial angular speed and the angle through which the wheel rotated. Use I and α together in equation 11-4 to find the torque exerted on the wheel.

Solution: 1. (a) Use Table 10-1 to find $I = \frac{1}{2}MR^2$: $I = \frac{1}{2}MR^2 = \frac{1}{2}(6.4\text{ kg})(0.71\text{ m})^2 = \underline{\underline{1.6\text{ kg}\cdot\text{m}^2}}$

2. Solve equation 10-11 for α: $\alpha = \dfrac{\omega^2 - \omega_0^2}{2\Delta\theta} = \dfrac{0^2 - (1.22\text{ rad/s})^2}{2(0.75\text{ rev}\times 2\pi\text{ rad/rev})} = \underline{\underline{-0.158\text{ rad/s}^2}}$

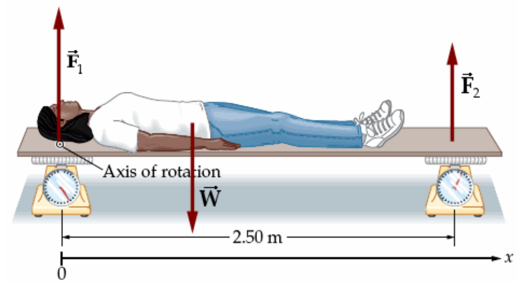
3. Apply equation 11-4 directly: $\tau = I\alpha = (1.6\text{ kg}\cdot\text{m}^2)(-0.158\text{ rad/s}^2) = \boxed{-0.25\text{ N}\cdot\text{m}}$

4. (b) If the mass of the wheel is doubled and its radius is halved, the moment of inertial will be cut in half (doubled because of the mass, cut to a fourth because of the radius). Therefore the magnitude of the angular acceleration will increase if the frictional torque remains the same, and the angle through which the wheel rotates before coming to rest will $\boxed{\text{decrease}}$.

Insight: If the moment of inertia is cut in half, the angular acceleration will double to -0.32 rad/s^2 and the angle through which the wheel rotates will be cut in half to 0.38 rev. This is because the wheel has less rotational inertia but the frictional torque remains the same. We bent the rules for significant figures in step 2 to avoid rounding error in step 3.

26. **IP BIO A Person's Center of Mass** To determine the location of her center of mass, a physics student lies on a lightweight plank supported by two scales 2.50 m apart, as indicated in **Figure 11–26xx31**. If the left scale reads 290 N, and the right scale reads 122 N, find **(a)** the student's mass and **(b)** the distance from the student's head to her center of mass.

Picture the Problem: The person lies on a lightweight plank that rests on two scales as shown in the diagram at right.



Strategy: Write Newton's Second Law in the vertical direction and Newton's Second Law for rotation to obtain two equations with two unknowns, m and x_{cm}. Solve each to find m and x_{cm}. Using the left side of the plank as the origin, there are two torques to consider: the positive torque due to the right hand scale and the negative torque due to the person's mass.

Solution: 1. (a) Write Newton's Second Law in the vertical direction to find *m*:

$$\sum F_y = F_1 + F_2 - mg = 0$$

$$m = \frac{F_1 + F_2}{g} = \frac{290 + 122 \text{ N}}{9.81 \text{ m/s}^2} = \boxed{42 \text{ kg}}$$

2. (b) Write Newton's Second Law for rotation and solve for x_{cm}:

$$\sum \tau = r_2 F_2 - x_{cm} mg = 0$$

$$x_{cm} = \frac{x_2 F_2}{mg} = \frac{(2.50 \text{ m})(122 \text{ N})}{(42 \text{ kg})(9.81 \text{ m/s}^2)} = \boxed{0.74 \text{ m}}$$

Insight: The equation in step 1 does not depend on the axis of rotation that we choose, but the equation in step 2 does. Nevertheless, we find exactly the same x_{cm} if we choose the other scale, near her feet, to be the axis of rotation.

49. **IP** You pull downward with a force of 28 N on a rope that passes over a disk-shaped pulley of mass 1.2 kg and radius 0.075 m. The other end of the rope is attached to a 0.67-kg mass. **(a)** Is the tension in the rope the same on both sides of the pulley? If not, which side has the largest tension? **(b)** Find the tension in the rope on both sides of the pulley.

Picture the Problem: You pull straight downward on a rope that passes over a disk-shaped pulley and then supports a weight on the other side. The force of your pull rotates the pulley and accelerates the mass upward.

Strategy: Write Newton's Second Law for the hanging mass and Newton's Second Law for torque about the axis of the pulley, and solve the two expressions for the tension T_2 at the other end of the rope. We are given in the problem that $T_1 = 25$ N. Let *m* be the mass of the pulley, *r* be the radius of the pulley, and *M* be the hanging mass. For the disk-shaped pulley the moment of inertia is $I = \frac{1}{2}mr^2$.

Solution: 1. (a) $\boxed{\text{No}}$, the tension in the rope on the other end of the rope accelerates the hanging mass, but the tension on your side both imparts angular acceleration to the pulley and accelerates the hanging mass. Therefore, the rope on $\boxed{\text{your side}}$ of the pulley has the greater tension.

2. (b) As stated in the problem, $T_1 = \boxed{28 \text{ N}}$ for the rope on your side of the pulley.

3. Set $\sum \vec{\mathbf{F}} = m\vec{\mathbf{a}}$ for the hanging mass: $\sum F_y = T_2 - Mg = Ma$

4. Set $\sum \tau = I\alpha$ for the pulley: $\sum \tau = rT_1 - rT_2 = I\alpha = \left(\frac{1}{2}mr^2\right)(a/r) \Rightarrow a = 2(T_1 - T_2)/m$

5. Substitute the expression for *a* from step 4 into the one from step 3, and solve for T_2 (the tension on the other side of the pulley from you):

$$T_2 - Mg = M\left[2(T_1 - T_2)/m\right]$$

$$mT_2 - mMg = 2MT_1 - 2MT_2$$

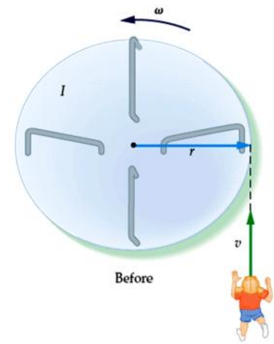
$$T_2 = \frac{M(2T_1 + mg)}{2M + m}$$

$$= \frac{(0.67 \text{ kg})\left[2(28 \text{ N}) + (1.2 \text{ kg})(9.81 \text{ m/s}^2)\right]}{2(0.67 \text{ kg}) + 1.2 \text{ kg}} = \boxed{18 \text{ N}}$$

Insight: The net force on the hanging mass is thus $T_2 - Mg = 18 - 6.6 \text{ N} = 11.4 \text{ N}$, enough to accelerate it upward at 17 m/s². The angular acceleration of the pulley is thus $a/r = (17 \text{ m/s}^2)/(0.075 \text{ m}) = 230 \text{ rad/s}^2$.

69. A disk-shaped merry-go-round of radius 2.63 m and mass 155 kg rotates freely with an angular speed of 0.641 rev/s. A 59.4-kg person running tangential to the rim of the merry-go-round at 3.41 m/s jumps onto its rim and holds on. Before jumping on the merry-go-round, the person was moving in the same direction as the merry-go-round's rim. What is the final angular speed of the merry-go-round?

Picture the Problem: A child runs tangentially to a rotating merry-go-round and hops on.



Strategy: Use conservation of angular momentum because there is no net torque on the system as long as the system includes both the person and the merry-go-round. Find the moments of inertia of the disk-shaped merry-go-round, $I_{\text{mgr}} = \frac{1}{2}M\,r^2$, and the system after the person hops on $I_{\text{f}} = \frac{1}{2}M\,r^2 + m\,r^2$, where M is the mass of the merry-go-round, m is the mass of the person, and r is the radius of the merry-go-round. Set $L_{\text{i}} = L_{\text{f}}$ and solve for the final angular speed ω_{f}, where the initial angular speed is:
$\omega_{\text{i}} = (0.641 \text{ rev/s})(2\pi \text{ rad/rev}) = 4.03 \text{ rad/s}.$

Solution: 1. Set $L_{\text{i}} = L_{\text{f}}$ and rearrange

$$L_{\text{disk}} + L_{\text{person}} = L_{\text{final}}$$

$$\left(\tfrac{1}{2}M\,r^2\right)\omega_{\text{i}} + mv\,r = \left(\tfrac{1}{2}M\,r^2 + m\,r^2\right)\omega_{\text{f}}$$

2. Now solve for ω_{f}:

$$\omega_{\text{f}} = \frac{\frac{1}{2}M\,r^2\omega_{\text{i}} + mv\,r}{\frac{1}{2}M\,r^2 + m\,r^2} = \frac{Mr\omega_{\text{i}} + 2mv}{M\,r + 2m\,r}$$

$$= \frac{(155\text{ kg})(2.63\text{ m})(4.03\text{ rad/s}) + 2(59.4\text{ kg})(3.41\text{ m/s})}{(155\text{ kg})(2.63\text{ m}) + 2(59.4\text{ kg})(2.63\text{ m})} = \boxed{2.84\text{ rad/s}}$$

Insight: The merry-go-round has slowed down because the initial linear speed of the person (3.41 m/s) is less than the initial linear speed of the rim of the merry-go-round (10.6 m/s).

79. A person exerts a tangential force of 36.1 N on the rim of a disk-shaped merry-go-round of radius 2.74 m and mass 167 kg. If the merry-go-round starts at rest, what is its angular speed after the person has rotated it through an angle of 32.5°

Picture the Problem: The merry-go-round is a uniform disk that is given an angular acceleration about its center of mass by the application of an unbalanced torque.

Strategy: The work done by the applied torque imparts kinetic energy to the merry-go-round. Set the torque times the angular displacement equal to the final kinetic energy of the merry-go-round (equations 11-17 and 11-18) and solve for ω_{f}. The moment of inertia of the merry-go-round is taken to be $I = \frac{1}{2}MR^2$, as indicated in Table 10-1 for a uniform disk rotating about its axis.

Solution: 1. Set $W = \Delta K$, applying equations 11-17, 11-3, and 10-17:

$$\tau\,\Delta\theta = (rF)\,\Delta\theta = K_{\text{f}} - K_{\text{i}} = \tfrac{1}{2}I\omega_{\text{f}}^2 - 0$$

2. Solve for ω_{f}:

$$\omega_{\text{f}} = \sqrt{\frac{2\,rF\,\Delta\theta}{I}} = \sqrt{\frac{2\,RF\,\Delta\theta}{\frac{1}{2}MR^2}}$$

$$= \sqrt{\frac{2(2.74\text{ m})(36.1\text{ N})(32.5°\times\pi\text{ rev}/180°)}{\frac{1}{2}(167\text{ kg})(2.74\text{ m})^2}} = \boxed{0.423\text{ rad/s}}$$

Insight: This rotation rate corresponds to a linear speed of only 1.16 m/s for the rim of the merry-go-round. The applied force did 56.1 J of work to give the merry-go-round 56.1 J of rotational kinetic energy.