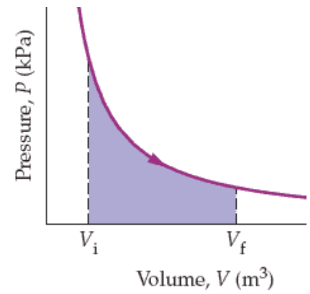
Discussion Examples Chapter 18: The Laws of Thermodynamics

20. **IP** If 8.00 moles of a monatomic ideal gas at a temperature of 245 K are expanded isothermally from a volume of 1.12 L to a volume of 4.33 L, calculate **(a)** the work done and **(b)** the heat flow into or out of the gas. **(c)** If the number of moles is doubled, by what factors do your answers to parts (a) and (b) change? Explain.

Picture the Problem: A monatomic ideal gas expands at constant temperature.



Strategy: Use equation 18-5 to find the work done in the isothermal process. Then solve the first law of thermodynamics (equation 18-3) for the heat.

Solution: 1. (a) Apply equation 18-5 directly: $W = nRT\ln\left(\frac{V_f}{V_i}\right)$

2. Insert the given data: $W = 8.00\text{ mol}\left[8.31\text{ J/}(\text{mol}\cdot\text{K})\right](245\text{ K})\ln\left(\frac{4.33\text{L}}{1.12\text{L}}\right) = \boxed{22.0\text{ kJ}}$

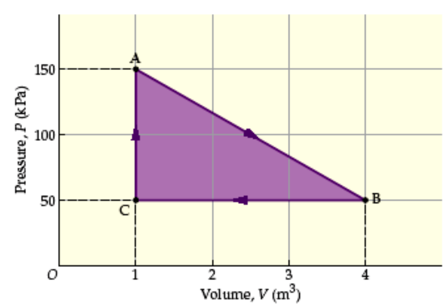
3. (b) Solve equation 18-3for Q: $Q = \Delta U + W = 0 + 22.0\text{ kJ} = \boxed{22.0\text{ kJ}}$ The heat flow is positive so this is heat flow $\boxed{\text{into the gas}}$.

4. (c) Both answers $\boxed{\text{increase by a factor of 2}}$ because the work done is proportional to number of moles of gas.

Insight: The internal energy of a monatomic ideal gas depends only on the temperature of the gas. In an isothermal process the temperature remains constant, and therefore so does the internal energy.

27. **IP** Suppose 67.5 moles of an ideal monatomic gas undergo the series of processes shown in Figure 18–24. **(a)** Calculate the temperature at the points A, B, and C. **(b)** For each process, A→B, B→C, and C→A, state whether heat enters or leaves the system. Explain in each case. **(c)** Calculate the heat exchanged with the gas during each of the three processes.

Picture the Problem: An ideal gas completes a three-process cycle.



Strategy: Use the ideal gas law to calculate the temperature at each state. Solve the first law of thermodynamics for the heat absorbed in each process.

Solution: 1. (a) Solve the ideal gas law for temperature: $T = \frac{PV}{nR}$

2. Solve for T_A: $T_A = \frac{(150\times10^3\text{ Pa})(1.00\text{ m}^3)}{(67.5\text{ mol})(8.31\text{ J/mol}\cdot\text{K})} = \boxed{267\text{ K}}$

3. Solve for T_B: $T_B = \frac{(50\times10^3\text{ Pa})(4.00\text{ m}^3)}{(67.5\text{ mol})(8.31\text{ J/mol}\cdot\text{K})} = \boxed{357\text{ K}}$

4. Solve for T_C: $T_C = \frac{(50\times10^3\text{ Pa})(1.00\text{ m}^3)}{(67.5\text{ mol})(8.31\text{ J/mol}\cdot\text{K})} = \boxed{89.1\text{ K}}$

5. (b) $A \to B$: The temperature rises and the gas does work, so heat $\boxed{\text{enters}}$ the system.
$B \to C$: The temperature drops and work is done on the gas, so heat $\boxed{\text{leaves}}$ the system.
$C \to A$: The temperature rises and no work is done on or by the gas, so heat $\boxed{\text{enters}}$ the system.

6. (c) Solve the first law for Q: $Q = \Delta U + W = \frac{3}{2}nR\Delta T + W$

7. Insert values for $\mathrm{A} \to \mathrm{B}$:

$$Q = \tfrac{3}{2}(67.5\ \text{mol})(8.31\ \text{J/mol}\cdot\text{K})(357\ \text{K} - 267\ \text{K}) + \tfrac{1}{2}(150\ \text{kPa} + 50\ \text{kPa})(3.00\ \text{m}^3) = \boxed{376\ \text{kJ}}$$

8. Insert values for $\mathrm{B} \to \mathrm{C}$:

$$Q = \tfrac{3}{2}(67.5\ \text{mol})(8.31\ \text{J/mol}\cdot\text{K})(89.1\ \text{K} - 357\ \text{K}) + 50\ \text{kPa}(-3.00\ \text{m}^3) = \boxed{-375\ \text{kJ}}$$

9. Insert values for $\mathrm{C} \to A$:

$$Q = \tfrac{3}{2}(67.5\ \text{mol})(8.31\ \text{J/mol}\cdot\text{K})(267\ \text{K} - 89.1\ \text{K}) + 0 = \boxed{150\ \text{kJ}}$$

Insight: The net work done during this cycle is equal to the net heat absorbed, or $W = 150$ kJ. Although the sum of the heats in part (d) appears to be 151 kJ, that is an artifact of significant digits and rounding issues, it's exactly 150 kJ.

30. **IP** An ideal monatomic gas is held in a perfectly insulated cylinder fitted with a movable piston. The initial pressure of the gas is 110 kPa, and its initial temperature is 280 K. By pushing down on the piston, you are able to increase the pressure to 140 kPa. **(a)** During this process, did the temperature of the gas increase, decrease, or stay the same? Explain. **(b)** Find the final temperature of the gas.

Picture the Problem: A monatomic ideal gas is adiabatically compressed.

Strategy: Use the ideal gas law to relate the initial and final conditions of the gas. Then use equation 18-9 to eliminate the unknown volumes from the equation.

Solution: 1 (a) Doing work on the system must $\boxed{\text{increase}}$ the internal energy and therefore the temperature when no heat flows into or out of the system.

2. (b) Write the ideal gas law in terms of the initial and final states:

$$PV = nRT \quad \Rightarrow \quad \frac{P_\mathrm{i} V_\mathrm{i}}{T_\mathrm{i}} = nR = \frac{P_\mathrm{f} V_\mathrm{f}}{T_\mathrm{f}}$$

3. Solve for the final temperature:

$$T_\mathrm{f} = \left(\frac{P_\mathrm{f}}{P_\mathrm{i}}\right)\left(\frac{V_\mathrm{f}}{V_\mathrm{i}}\right) T_\mathrm{i}$$

4. Now solve equation 18-9 for the ratio of volumes:

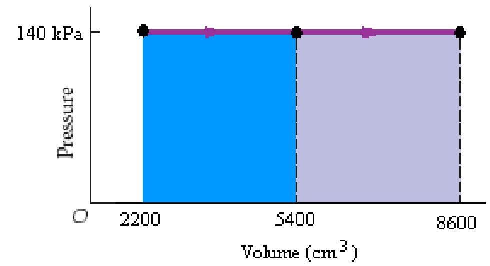
$$P_i V_i^\gamma = P_f V_f^\gamma \quad \Rightarrow \quad \frac{V_\mathrm{f}}{V_\mathrm{i}} = \left(\frac{P_\mathrm{i}}{P_\mathrm{f}}\right)^{\frac{1}{\gamma}} = \left(\frac{P_\mathrm{f}}{P_\mathrm{i}}\right)^{-\frac{1}{\gamma}}, \text{ where } \gamma = \tfrac{5}{3}$$

5. Insert this ratio into the equation for T_f:

$$T_\mathrm{f} = T_\mathrm{i}\left(\frac{P_\mathrm{f}}{P_\mathrm{i}}\right)\left(\frac{P_\mathrm{f}}{P_\mathrm{i}}\right)^{-\frac{1}{\gamma}} = T_\mathrm{i}\left(\frac{P_\mathrm{f}}{P_\mathrm{i}}\right)^{1-\frac{1}{\gamma}} = (280\ \text{K})\left(\frac{140\ \text{kPa}}{110\ \text{kPa}}\right)^{\left(1-\frac{3}{5}\right)} = \boxed{310\ \text{K}}$$

Insight: As predicted, when work was done on the gas in an adiabatic process the temperature increased.

38. **IP** A cylinder contains 18 moles of a monatomic ideal gas at a constant pressure of 160 kPa. **(a)** How much work does the gas do as it expands 3200 cm^3, from 5400 cm^3 to 8600 cm^3? **(b)** If the gas expands by 3200 cm^3 again, this time from 2200 cm^3 to 5400 cm^3, is the work it does greater than, less than, or equal to the work found in part (a)? Explain. **(c)** Calculate the work done as the gas expands from 2200 cm^3 to 5400 cm^3.



Picture the Problem: A monatomic ideal gas is expanded at constant pressure by a fixed change in volume. This process is done twice with differing initial volumes.

Strategy: Use the area under the *PV* plot to calculate the work done in each process.

Solution: 1. (a) Write the work as area under the *PV* plot: $W = P\Delta V$

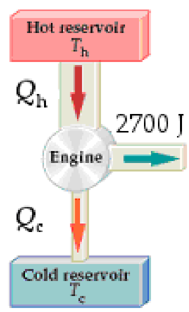
2. Solve numerically: $W = (160\times10^3\ \text{Pa})(8600\ \text{cm}^3 - 5400\ \text{cm}^3)\left(\dfrac{1\times10^{-6}\ \text{m}^3}{\text{cm}^3}\right) = \boxed{0.51\ \text{kJ}}$

3. (b) Work is directly proportional to the change in volume. Therefore, the work done by the gas in the second expansion is equal to that done in the first expansion.

4. (c) Solve numerically: $W = (160\times10^3\ \text{Pa})(5400\ \text{cm}^3 - 2200\ \text{cm}^3)\left(\dfrac{1\times10^{-6}\ \text{m}^3}{\text{cm}^3}\right) = \boxed{0.51\ \text{kJ}}$

Insight: Because the work is proportional to the change in volume, increasing the volume by 3200 cm^3 from any initial volume will produce the same amount of work.

50. **IP** If a heat engine does 2700 J of work with an efficiency of 0.18, find **(a)** the heat taken in from the hot reservoir and **(b)** the heat given off to the cold reservoir. **(c)** If the efficiency of the engine is increased, do your answers to parts (a) and (b) increase, decrease, or stay the same? Explain.



Picture the Problem: A heat engine produces 2700 J of work as it extracts heat from a hot temperature reservoir and rejects heat to a cold temperature reservoir.

Strategy: Use equation 18-11 to calculate Q_h and then equation 18-10 to calculate Q_c :

Solution: 1. (a) Solve equation 18-11 for Q_h : $Q_h = \dfrac{W}{e}$

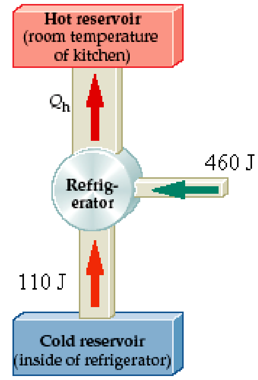
2. Insert the given values: $Q_h = \dfrac{2700\ \text{J}}{0.18} = \boxed{15\ \text{kJ}}$

3. (b): Solve equation 18-19 for Q_c : $Q_c = Q_h - W = 1.50\times10^4\ \text{J} - 2700\ \text{J} = \boxed{12\ \text{kJ}}$

4. (c) Higher efficiency means less heat input is needed to produce the same work. Consequently, less heat is lost to the surroundings. The answers in parts (a) and (b) will decrease.

Insight: As an example of higher efficiency, if $e = 0.24$ and $W = 2700$ J, then $Q_h = 10.0$ kJ and $Q_c = 7.6$ kJ.

56. The refrigerator in your kitchen does 480 J of work to remove 110 J of heat from its interior. **(a)** How much heat does the refrigerator exhaust into the kitchen? **(b)** What is the refrigerator's coefficient of performance?



Picture the Problem: A refrigerator extracts heat from the cold reservoir and ejects the heat to the hot reservoir. This process requires work to be input into the system.

Strategy: Solve equation 18-10 for the heat exhausted to the hot reservoir. Then use equation 18-15 to solve for the coefficient of performance of the refrigerator.

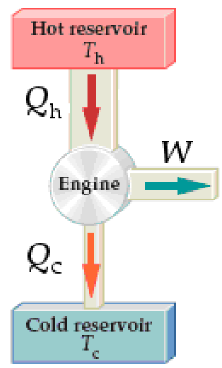
Solution: 1. (a) Solve equation 18-10 for Q_h: $Q_h = Q_c + W$

2. Insert the given values: $Q_h = 110\text{ J} + 480\text{ J} = \boxed{0.59\text{ kJ}}$

3. (b) Insert the values into equation 18-15: $\text{COP} = \frac{Q_c}{W} = \frac{110\text{ J}}{480\text{ J}} = \boxed{0.23}$

Insight: This rather inefficient refrigerator exhausts to the room over five times the amount of heat extracted from inside the refrigerator. The additional heat comes from the work done by the refrigerator.

72. A heat engine operates between a high-temperature reservoir at 610 K and a low-temperature reservoir at 320 K. In one cycle, the engine absorbs 6400 J of heat from the high-temperature reservoir and does 2200 J of work. What is the net change in entropy as a result of this cycle?



Picture the Problem: A heat engine extracts 6400 J from a hot reservoir and performs 2200 J of work. The remainder of the heat is exhausted to the cold reservoir.

Strategy: Calculate the net change in entropy by summing the changes in entropy at the hot and cold reservoirs. Use equation 18-18 to write the entropy change in terms of the heat transfers and temperature. Finally, use equation 18-10 to find the heat exchange at the cold reservoir.

Solution: 1. Sum the entropy changes: $\Delta S = \left(\frac{Q}{T}\right)_h + \left(\frac{Q}{T}\right)_c$

2. Write in terms from the hot and cold reservoirs: $\Delta S = \left(\frac{-Q_h}{T_h}\right) + \left(\frac{Q_c}{T_c}\right)$

3. Use equation 18-10 to eliminate Q_c: $\Delta S = \left(\frac{-Q_h}{T_h}\right) + \left(\frac{Q_h - W}{T_c}\right)$

4. Substitute numerical values: $\Delta S = \frac{-6400\text{ J}}{610\text{ K}} + \frac{6400\text{ J} - 2200\text{ J}}{320\text{ K}} = \boxed{2.6\text{ J/K}}$

Insight: This engine operates at less than maximum efficiency. The Carnot efficiency of this engine is 0.475 and the actual efficiency is 0.344. If the engine were running at maximum efficiency the net change in entropy would be zero.