Discussion Examples Chapter 15: Fluids

8. A 79-kg person sits on a 3.7-kg chair. Each leg of the chair makes contact with the floor in a circle that is 1.3 cm in diameter. Find the pressure exerted on the floor by each leg of the chair, assuming the weight is evenly distributed.

Picture the Problem: When a person sits in a four-legged chair the weight of the person and chair is distributed over each leg of the chair, increasing the pressure each leg exerts on the ground.

Strategy: Use equation 15-2 to calculate the pressure each leg exerts on the floor. Set the force equal to the sum of the weights of the person and chair and the area equal to four times the cross-sectional area of each leg.

Solution: 1. Set the pressure equal to the weight divided by area:

$$P = \frac{F}{A} = \frac{mg}{4\left[\pi\left(\frac{d}{2}\right)^2\right]} = \frac{mg}{\pi d^2}$$

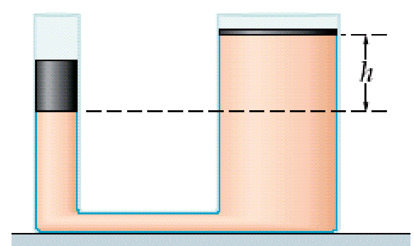
2. Insert given data:

$$P = \frac{(79\text{ kg} + 3.7\text{ kg})(9.81\text{ m/s}^2)}{\pi\ (0.013\text{ m})^2} = \boxed{1.5\times 10^6\text{ Pa}} = 1.5\text{ MPa}$$

Insight: Leaning back in the chair, so that is rests on only two legs, doubles the pressure those legs exert on the floor.

18. In the hydraulic system shown in the figure, the piston on the left has a diameter of 4.4 cm and a mass of 1.8 kg. The piston on the right has a diameter of 12 cm and a mass of 3.2 kg. If the density of the fluid is 750 kg/m^3, what is the height difference h between the two pistons?

Picture the Problem: Two pistons are supported by a fluid, as shown in the figure. The pressure in the fluid at the bottom of the left piston is equal to the pressure in the right piston at the same vertical level, which is a distance h below the right piston.



Strategy: Set the pressures in the two columns equal at the depth h. Use equation 15-1 to calculate the pressure due to the pistons and equation 15-7 to calculate the increase in pressure due to the fluid in the right-hand column.

Solution: 1. Set the pressures equal:

$$P_L = P_R$$

$$\frac{m_L g}{A_L} = \frac{m_R g}{A_R} + \rho g h$$

2. Solve for the height h:

$$h = \frac{1}{\rho}\left(\frac{m_L}{A_L} - \frac{m_R}{A_R}\right) = \frac{1}{\rho}\left(\frac{m_L}{\frac{\pi}{4}D_L^2} - \frac{m_R}{\frac{\pi}{4}D_R^2}\right) = \frac{4}{\pi\rho}\left(\frac{m_L}{D_L^2} - \frac{m_R}{D_R^2}\right)$$

3. Insert the given values:

$$h = \frac{4}{\pi(750\text{kg/m}^3)}\left[\frac{1.8\text{ kg}}{(0.044\text{ m})^2} - \frac{3.2\text{ kg}}{(0.12\text{ m})^2}\right] = \boxed{1.2\text{ m}}$$

Insight: The distance h does not depend on the overall height of the pistons.

34. A 3.2-kg balloon is filled with helium $(\text{density} = 0.179\ \text{kg/m}^3)$. If the balloon is a sphere with a radius of 4.9 m, what is the maximum weight it can lift?

Picture the Problem: A helium balloon displaces heavier air, causing it to rise. We want to calculate the maximum additional weight that the balloon can lift.

Strategy: Calculate the buoyant force on the balloon using equation 15-9 with the density of air and the volume of a sphere. Subtract the weight of the balloon and the weight of the helium ($W = \rho_{He} V g$) to calculate the additional weight the balloon can lift.

Solution: 1. Calculate the buoyant force on the balloon:

$$F_\text{b} = \rho_\text{air} V g = \rho_\text{air}\left[\frac{4}{3}\pi r^3\right] g = \left(1.29\ \text{kg/m}^3\right)\left[\frac{4}{3}\pi\left(4.9\,\text{m}\right)^3\right]\left(9.81\ \text{m/s}^2\right) = \underline{\underline{6236\,\text{N}}}$$

2. Subtract the weight of the balloon and of the helium:

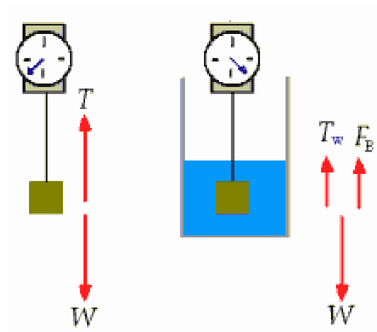
$$W = F_\text{b} - m_\text{balloon} g - \rho_\text{He} V_\text{He} g = 6236\ \text{N} - \left(3.2\ \text{kg}\right)\left(9.81\ \text{m/s}^2\right)$$

$$-\left(0.179\ \text{kg/m}^3\right)\left[\frac{4}{3}\pi\left(4.9\ \text{m}\right)^3\right]\left(9.81\ \text{m/s}^2\right) = \boxed{5.3\ \text{kN}}$$

Insight: In this problem the weight of the balloon (31 N) is negligible. The additional mass that the balloon can lift is equal to the difference in densities of the air and helium times the volume of the balloon.

43. A solid block is attached to a spring scale. When the block is suspended in air, the scale reads 20.0 N; when it is completely immersed in water, the scale reads 17.7 N. What is **(a)** the volume and **(b)** the density of the block?

Picture the Problem: When a block is suspended from a scale, its weight is equal to the tension on the scale. When the block is suspended in water, its weight is equal to the sum of the tension and buoyant force, as shown in the figure.



Strategy: We wish to calculate the volume and density of the block from the scale readings. Solve Newton's Second Law for the mass suspended in the water to determine the volume of the block. The weight of the block is given from the scale when the block is suspended in air. The density of water is given in Table 15-1. Calculate the density by dividing the block's mass by the volume.

Solution: 1. (a) Solve Newton's Second Law for the buoyant force:

$$\sum F = T_w + F_B - W = 0$$

$$F_B = \rho_\text{w} V g = W - T_w$$

2. Solve for the block's volume:

$$V = \frac{W - T_\text{w}}{\rho_\text{w} g} = \frac{20.0\ \text{N} - 17.7\ \text{N}}{\left(1000\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)} = \boxed{2.34\times 10^{-4}\ \text{m}^3}$$

3. (b) Calculate the density using equation 15-1:

$$\rho = \frac{m}{V} = \frac{W}{gV} = \frac{20.0\ \text{N}}{\left(9.81\ \text{m/s}^2\right)\left(2.345\times 10^{-4}\ \text{m}^3\right)} = \boxed{8.70\times 10^3\ \text{kg/m}^3}$$

Insight: Comparing the submerged and non-submerged weights of an irregularly shaped object is an effective way of determining its volume and density.

47. A person with a mass of 81 kg and a volume of $0.089\ \text{m}^3$ floats quietly in water. **(a)** What is the volume of the person that is above water? **(b)** If an upward force F is applied to the person by a friend, the volume of the person above water increases by $0.0018\ \text{m}^3$. Find the force F.

Picture the Problem: A person is floating in water. Because the person is not accelerating, the buoyant force must equal her weight. When an upward force is applied to the swimmer, her body rises out of the water such that the new buoyant force and the applied force are equal to her weight.

Strategy: Set the buoyant force (equation 15-9) equal to the person's weight and solve for the submerged volume. Subtract the submerged volume from the total volume to calculate the volume above the surface. Set the sum of the buoyant force and the applied force equal to the weight and solve for the applied force.

Solution: 1. (a) Set the buoyant force equal to the weight and solve for volume submerged:

$$F_b = W$$

$$\rho_w V_{sub} g = mg \quad \Rightarrow \quad V_{sub} = \frac{m}{\rho_w}$$

2. Subtract submerged volume from total volume:

$$V_{above} = V_{total} - V_{sub} = V_{total} - \frac{m}{\rho_w}$$

$$= 0.089\ \text{m}^3 - \frac{81\ \text{kg}}{1000\ \text{kg/m}^3} = \boxed{0.008\ \text{m}^3}$$

3. (b) Set the applied force and buoyant force equal to the weight and solve for the applied force:

$$F_{applied} + F_b = W$$

$$F_{applied} = W - F_b = mg - \rho_w V_{sub2} g$$

4. Set the submerged volume equal to the initially submerged volume minus the given change in volume:

$$F_{applied} = mg - \rho_w \left(V_{sub} - 0.0018\ \text{m}^3\right) g$$

$$= mg - \rho_w \left(\frac{m}{\rho_w} - 0.0018\ \text{m}^3\right) g$$

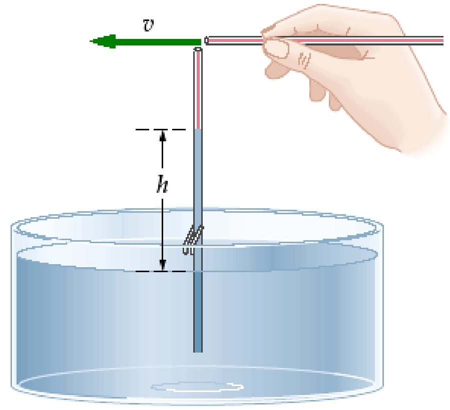
$$= \rho_w \left(0.0018\ \text{m}^3\right) g$$

$$F_{applied} = \left(1000\ \text{kg/m}^3\right)\left(0.0018\ \text{m}^3\right)\left(9.81\ \text{m/s}^2\right) = \boxed{18\ \text{N}}$$

Insight: Note that the applied force equals the change in the buoyant force on the person, that is the density of the water times the change in volume submerged times gravity.

61. **BIO A Blowhard** Tests of lung capacity show that adults are able to exhale 1.5 liters of air through their mouths in as little as 1.0 seconds. **(a)** If a person blows air at this rate through a drinking straw with a diameter of 0.60 cm, what is the speed of air in the straw? **(b)** If the air from the straw in part (a) is directed horizontally across the upper end of a second straw that is vertical, as shown in the figure, to what height does water rise in the vertical straw?

Picture the Problem: As shown in the figure, a person blows air through the straw at 1.5 liters per second. As the air passes over the vertical straw it decreases the pressure in the straw, which causes the water to rise. We wish to calculate the speed at which the air exits the straw and the height to which the water rises.



Strategy: Divide the volume flow rate of the air by the cross-sectional area of the straw to calculate the speed of the air. Use equation 15-14 to calculate the difference in pressure between point 1 at the top of the vertical straw and point 2 out in the room. Calculate the height of the water column from the pressure variation with depth (equation 15-7) using the pressure difference between the top of the straw and the room.

Solution: 1. (a) Solve the volume flow rate for the air velocity:

$$\frac{\Delta V}{\Delta t} = Av \Rightarrow v = \frac{\Delta V/\Delta t}{\left(\frac{1}{4}\pi d^2\right)} = \frac{\left(1.5\text{ L/s}/1000\text{ L/m}^3\right)}{\frac{1}{4}\pi\left(0.0060\text{ m}\right)^2} = \boxed{53\text{ m/s}}$$

2. (b) Solve equation 15-14 for the pressure difference:

$$P_1 + \tfrac{1}{2}\rho v_1^2 = P_2 + \tfrac{1}{2}\rho v_2^2$$

$$P_2 - P_1 = \Delta P = \tfrac{1}{2}\rho v_1^2 = \tfrac{1}{2}\left(1.29\text{ kg/m}^3\right)\left(53.1\text{ m/s}\right)^2 = \underline{\underline{1.82\text{ kPa}}}$$

3. Solve equation 15-7 for the height of the water:

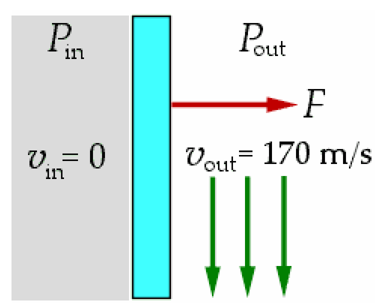
$$P_2 = P_1 + \rho_w hg$$

$$h = \frac{P_2 - P_1}{\rho_w g} = \frac{\Delta P}{\rho_w g} = \frac{1.815\text{ kPa}}{\left(1000\text{ kg/m}^3\right)\left(9.81\text{ m/s}^2\right)} = \boxed{19\text{ cm}}$$

Insight: The height of the water column is inversely proportional to the fourth power of the straw's diameter. If the diameter of the straw were decreased by a factor of two (d = 0.30 cm) the height of the water column would increase to 2.91 m, or 2^4=16 times the height in the problem.

66. On a vacation flight, you look out the window of the jet and wonder about the forces exerted on the window. Suppose the air outside the window moves with a speed of approximately 170 m/s shortly after takeoff, and that the air inside the plane is at atmospheric pressure. **(a)** Find the pressure difference between the inside and outside of the window. **(b)** If the window is 25 cm by 42 cm, find the force exerted on the window by air pressure.

Picture the Problem: The figure shows an airplane window. Air rushes past the outside of the window at 170 m/s, while the air inside is at rest.



Strategy: We wish to calculate the pressure difference between inside and outside and the net force on the window. Solve equation 15-14, Bernoulli's equation, for the pressure difference. Multiply the pressure difference by the area of the window to calculate the net force.

Solution: 1. (a) Solve Bernoulli's equation for the pressure difference:

$$P_{in} + \tfrac{1}{2}\rho v_{in}^2 = P_{out} + \tfrac{1}{2}\rho v_{out}^2$$

$$P_{in} - P_{out} = \tfrac{1}{2}\rho\left(v_{out}^2 - v_{in}^2\right)$$

2. Insert the given values:

$$P_{in} - P_{out} = \tfrac{1}{2}\left(1.29\text{ kg/m}^3\right)\left[\left(170\text{ m/s}\right)^2 - 0^2\right] = \boxed{19\text{ kPa}}$$

3. (b) Multiply the pressure by the area of the window:

$$\Delta F = \Delta PA = 18.6\times10^3\text{ Pa}\left(0.25\text{ m}\right)\left(0.42\text{ m}\right) = \boxed{1.96\text{ kN}}$$

Insight: This pressure difference creates over 400 lbs of force on the window!