Discussion Examples
Chapter 12: Gravity

5. **The Attraction of Ceres** Ceres, the largest asteroid known, has a mass of roughly 8.7×10^{20} kg **.** If Ceres passes within 14,000 km of the spaceship in which you are traveling, what force does it exert on you? (Use an approximate value for your mass, and treat yourself and the asteroid as point objects.)

Picture the Problem: You and the asteroid attract each other gravitationally.

Strategy: Estimate that your mass $\approx$ 70 kg. Apply equation 12-1 to find the gravitational force between you and Ceres.

Solution: Apply equation 12-1:

$$F = G\frac{m_1 m_2}{r^2} = \left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\frac{\left(8.7\times10^{20}\ \text{kg}\right)\left(70\ \text{kg}\right)}{\left(14\times10^{6}\ \text{m}\right)^2} = \boxed{0.021\ \text{N}}$$

Insight: If you stood on the surface of Ceres (radius 500 km) the force would be 16 N (3.6 lb).

21. **IP An Extraterrestrial Volcano** Several volcanoes have been observed erupting on the surface of Jupiter's closest Galilean moon, Io. Suppose that material ejected from one of these volcanoes reaches a height of 5.00 km after being projected straight upward with an initial speed of 134 m/s. Given that the radius of Io is 1820 km, **(a)** outline a strategy that allows you to calculate the mass of Io. **(b)** Use your strategy to calculate Io's mass.

Picture the Problem: The volcano on Io spews material at high speed straight upward. The mass slows down, rising to a height of 5.00 km before coming to rest momentarily under the influence of Io's gravitation.

Strategy: Use conservation of energy to relate the initial kinetic energy of the ejected material to its potential energy at the maximum altitude. This relation will allow the calculation of the surface gravity of Io. Then use equation 12-4 and the given radius of Io to find the mass of Io.

Solution: 1. (a) $\boxed{\text{Use } \tfrac{1}{2}mv_i^2 = mgh_f \text{ to find } g, \text{ and use } g = GM/R^2 \text{ to find } M.}$

2. (b) Set $E_i = E_f$ and solve for g:

$$\tfrac{1}{2}mv_i^2 = mgh_f$$

$$g = \frac{v_i^2}{2h_f} = \frac{\left(134\ \text{m/s}\right)^2}{2\left(5.00\times10^3\ \text{m}\right)} = \underline{\underline{1.80\ \text{m/s}^2}}$$

3. Solve equation 12-4 for M:

$$M = \frac{gR^2}{G} = \frac{\left(1.80\ \text{m/s}^2\right)\left(1.82\times10^6\ \text{m}\right)^2}{6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2} = \boxed{8.94\times10^{22}\ \text{kg}}$$

Insight: The use of conservation of energy to find g is equivalent to solve equations 4-6, $v^2 = v_0^2 - 2g\Delta y$, for g. This approach assumes that g is constant over the 5.00 km that the ejected material rises. This is a pretty good assumption, because you will get the same answer to 3 significant figures if you apply the more exact method of section 12-5. The mass of Io according to NASA's Solar System Exploration web site is 8.9316×10^{22} kg.

22. **IP Verne's Trip to the Moon** In his novel *From the Earth to the Moon*, Jules Verne imagined that astronauts inside a spaceship would walk on the floor of the cabin when the force exerted on the ship by the Earth was greater than the force exerted by the Moon. When the force exerted by the Moon was greater, he thought the astronauts would walk on the ceiling of the cabin. **(a)** At what distance from the center of the Earth would the forces exerted on the spaceship by the Earth and the Moon be equal? **(b)** Explain why Verne's description of gravitational effects is incorrect.

Picture the Problem: Both the Earth and the Moon exert a gravitational force on the spaceship, but in opposite directions.

Strategy: Use the Universal Law of Gravity (equation 12-1) to relate the attractive forces from the Earth and the Moon. Set the force due to the Earth equal to the force due to the Moon when the spaceship is at a distance *r* from the center of the Earth. Let $R = 3.84\times10^8$ m, the distance between the centers of the Earth and Moon. Then solve the expression for the distance *r*.

Solution: 1. (a) Set $F_E = F_M$ using equation 12-1 and solve for *r*:

$$G\frac{m_s m_E}{r^2} = G\frac{m_s m_M}{(R-r)^2}$$

$$m_E (R-r)^2 = m_M r^2$$

$$R - r = \sqrt{m_M/m_E}\ r$$

$$r = \frac{R}{1+\sqrt{m_M/m_E}} = \frac{3.84\times10^8 \text{ m}}{1+\sqrt{(7.35\times10^{22}\text{ kg})/(5.97\times10^{24}\text{ kg})}}$$

$$= \boxed{3.46\times10^8 \text{ m}}$$

2. (b) The net gravitational force on the spaceship and the astronauts will steadily decrease, reaching zero at the location found in part (a), and then gradually increase in the opposite direction (toward the Moon). However, the astronauts are accelerated at the same rate as the spaceship and so they will appear to float, not walk.

Insight: The distance in part (a) is about 90% of the distance between the Earth and the Moon! The Earth's gravity is the dominant force through most of the journey to the Moon.

28. **Apollo Missions** On Apollo missions to the Moon, the command module orbited at an altitude of 110 km above the lunar surface. How long did it take for the command module to complete one orbit?

Picture the Problem: The Apollo capsule orbited the Moon at an altitude of 110 km above the Moon's surface.

Strategy: Use equation 12-7 to determine the period of orbit using the mass and radius of the Moon from the inside back cover of the text.

Solution: Apply equation 12-7:

$$T = \left(\frac{2\pi}{\sqrt{GM_M}}\right) r^{3/2}$$

$$= \left[\frac{2\pi}{\sqrt{(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(7.35\times10^{22}\text{ kg})}}\right](1.74\times10^6 + 110\times10^3 \text{ m})^{3/2}$$

$$T = 7140 \text{ s} = \boxed{1.98 \text{ h}}$$

Insight: This period turns out to be a bit larger than the 1.44-h orbit period of a satellite that is 110 km above the Earth's surface.

35. **IP** Two satellites orbit the Earth, with satellite 1 at a greater altitude than satellite 2. **(a)** Which satellite has the greater orbital speed? Explain. **(b)** Calculate the orbital speed of a satellite that orbits at an altitude of one Earth radius above the surface of the Earth. **(c)** Calculate the orbital speed of a satellite that orbits at an altitude of two Earth radii above the surface of the Earth.

Picture the Problem: The two satellites travel in circular orbits about the Earth.

Strategy: Set the gravitational force between the satellite and the Earth equal to the centripetal force required to keep the satellite moving in a circular path, and solve for the speed. Use equation 6-15, $f_{cp} = mv^2/r$, for the centripetal force. Use the mass and radius of the Earth given in the inside back cover of the text.

Solution: 1. (a) Kepler's Third Law (equation 12-6) indicates that the orbit period grows faster than the orbit radius. The orbit speed is proportional to the orbit radius divided by the period, and so the orbit speed is proportional to $v \propto r/T \propto r/r^{3/2} = 1/\sqrt{r}$. That means the orbit speed decreases as the radius increases, so $\boxed{\text{satellite 2}}$ will have the greater orbital speed.

2. (b) Set $F_{\text{gravity}} = F_{\text{centripetal}}$ and solve for v:

$$G\frac{M_E m}{(R_E + h)^2} = m\frac{v^2}{(R_E + h)}$$

$$v = \sqrt{\frac{GM_E}{R_E + h}} = \sqrt{\frac{GM_E}{R_E + R_E}} = \sqrt{\frac{(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(5.97\times10^{24}\ \text{kg})}{2\times6.37\times10^6\ \text{m}}}$$

$$= 5590\ \text{m/s} = \boxed{5.59\ \text{km/s}}$$

3. (c) Repeat step 2 for the new h:

$$v = \sqrt{\frac{GM_E}{R_E + h}} = \sqrt{\frac{GM_E}{R_E + 2R_E}} = \sqrt{\frac{(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(5.97\times10^{24}\ \text{kg})}{3\times6.37\times10^6\ \text{m}}}$$

$$= 4560\ \text{m/s} = \boxed{4.56\ \text{km/s}}$$

Insight: The satellite with the larger orbit radius in part (c) had the smaller orbit speed, as predicted.