Lecture 26
The Hertzsprung-Russell Diagram
January 13b, 2014
Hertzsprung-Russell Diagram

- Hertzsprung and Russell found a correlation between luminosity and spectral type (temperature)

Hertzsprung-Russell Diagram

Luminosity (Solar units)

- Hot, bright stars
- Sun
- Cool, dim stars
- Main Sequence

hot | O | B | A | F | G | K | M | cooler
• Most stars (~90%) are on the main sequence
  – The greater the temperature, the more luminous the star
  – M type stars are the most common.
  – O type stars are the least common.
Sizes of Stars

• Size of star is related to the temperature and the luminosity by:

\[ \text{Luminosity} \propto \text{Temperature}^4 \times \text{Radius}^2 \]

\[ L = \sigma T^4 \times 4\pi R^2 \]

– If you know the position on HR diagram you know the size of the star.
A star is twice as luminous as the Sun and has twice the surface temperature. What is its radius relative to the Sun’s radius?

A. \( \frac{R_{\text{star}}}{R_{\text{Sun}}} = 4.0 \)

B. \( \frac{R_{\text{star}}}{R_{\text{Sun}}} = 2.8 \)

C. \( \frac{R_{\text{star}}}{R_{\text{Sun}}} = 0.5 \)

D. \( \frac{R_{\text{star}}}{R_{\text{Sun}}} = 0.35 \)
A star is twice as luminous as the Sun and has twice the surface temperature. What is its radius relative to the Sun’s radius?

\[
L = \sigma T^4 4\pi R^2 \quad \Rightarrow \quad R = \sqrt{L/4\pi\sigma T^4}
\]

\[
\frac{R_{\text{star}}}{R_{\text{Sun}}} = \sqrt{\frac{L_{\text{star}}}{4\pi\sigma T_{\text{star}}^4}} = \frac{1}{\sqrt{\frac{L_{\text{Sun}}}{4\pi\sigma T_{\text{Sun}}^4}}} = \sqrt{\frac{L_{\text{star}}}{L_{\text{Sun}}} \left(\frac{T_{\text{Sun}}}{T_{\text{star}}}\right)^4}
\]

\[
= \sqrt{\frac{2 \left(\frac{1}{2}\right)^4}{1}} = 0.35
\]
Supergiants
Giants
Main Sequence

Luminosity (Solar units)

hotter
O B A F G K M cooler

Spectral Classification

0.1 R⊙
0.01 R⊙
0.0001
10000
100
100
1
0.01
0.0001

Cool, dim stars

Sun

White Dwarfs
• **Supergiants** -- cool, bright, red, large stars
• **Giants** -- cool, bright red, less large stars
• **Main Sequence** -- spans range from hot, bright stars to cool, dim stars.
• **White dwarfs** -- hot, small, dim stars.

• These classifications will give clues to stages in the evolution of stars.
Stars come in many sizes!
Sizes of Objects in our Universe

Images from webisto.com/space
Sizes of Objects in our Universe
Sizes of Objects in our Universe
Sizes of Objects in our Universe

- Sun
- Sirius
- Pollux
- Arcturus

Jupiter is about 1 pixel in size.

Earth is invisible at this scale.
Sizes of Objects in our Universe

- Betelgeuse
- Antares
- Sun (1 pixel)
- Jupiter is invisible at this scale
- Rigel
- Aldebaran
- Sirius
- Pollux
- Arcturus
Masses of Stars

• We cannot directly measure the mass of an isolated star.

• If something is orbiting the star, can use the general form of Kepler’s Third Law

\[
(M_1 + M_2)P^2 = a^3
\]

• Luckily, 2/3 of all stars are **binary stars**, two stars that orbit one another
A binary star system consists of one star that is twice as massive as the other. They are 2.0 AU apart and have an orbit period of 0.50 y. What is the mass of the smaller star in terms of solar masses?

A. $11 \ M_{\text{Sun}}$
B. $4 \ M_{\text{Sun}}$
C. $0.50 \ M_{\text{Sun}}$
D. $0.125 \ M_{\text{Sun}}$
A binary star system consists of one star that is twice as massive as the other. They are 2.0 AU apart and have an orbit period of 0.50 y. What is the mass of the smaller star in terms of solar masses?

A. $11 M_{\text{Sun}}$  
B. $4 M_{\text{Sun}}$  
C. $0.50 M_{\text{Sun}}$  
D. $0.125 M_{\text{Sun}}$  

\[
(m + 2m) P^2 = a^3
\]
\[
m = \frac{a^3}{3P^2} = \frac{(2.0 \text{ AU})^3}{3(0.50 \text{ y})^2} = 11M_{\odot}
\]
Mass-Luminosity Relation

- True ONLY for Main Sequence stars
- As the mass increases, the luminosity increases rapidly

\[ \text{Luminosity} \propto \text{Mass}^3 \]
HR Diagram: Main sequence stars labeled by mass in units of solar masses.
Question

How do the following properties differ with respect to the Sun for the listed main sequence types? What properties would not change for *non*-main sequence stars?

<table>
<thead>
<tr>
<th></th>
<th>O</th>
<th>G</th>
<th>M</th>
</tr>
</thead>
<tbody>
<tr>
<td>Temperature</td>
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<td></td>
<td></td>
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<tr>
<td>Mass</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Size</td>
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<tr>
<td>Color</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Luminosity</td>
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<td></td>
<td></td>
</tr>
</tbody>
</table>
Question

How do the following properties differ with respect to the Sun for the listed main sequence types? What properties would not change for non-main sequence stars?

<table>
<thead>
<tr>
<th>Property</th>
<th>O</th>
<th>G</th>
<th>M</th>
</tr>
</thead>
<tbody>
<tr>
<td>Temperature*</td>
<td>Higher</td>
<td>Same</td>
<td>Lower</td>
</tr>
<tr>
<td>Mass</td>
<td>Higher</td>
<td>Same</td>
<td>Lower</td>
</tr>
<tr>
<td>Size</td>
<td>Larger</td>
<td>Same</td>
<td>Smaller</td>
</tr>
<tr>
<td>Color*</td>
<td>Blue</td>
<td>White</td>
<td>Red</td>
</tr>
<tr>
<td>Luminosity</td>
<td>Higher</td>
<td>Same</td>
<td>Red</td>
</tr>
</tbody>
</table>
Apparent Brightness

• The brightness an object appears to have.
• The further away the object, the dimmer it looks

\[
\text{Apparent Brightness} = \frac{\text{Luminosity}}{4\pi d^2}
\]

\(d = \text{distance}\)
Star A has a brightness of $1.0 \, \mu\text{W/m}^2$ and is known to be 4.0 ly away. Star B is 9.0 ly away and has a brightness of $3.0 \, \mu\text{W/m}^2$. How much more luminous is star B than star A?

A. $240 \times$

B. $15 \times$

C. $6.8 \times$

D. $3.0 \times$
Star A has a brightness of $1.0 \, \mu W/m^2$ and is known to be 4.0 ly away. Star B is 9.0 ly away and has a brightness of $3.0 \, \mu W/m^2$. How much more luminous is star B than star A?

A. $240 \times$

B. $15 \times$

C. $6.8 \times$

D. $3.0 \times$

\[ L_B = \frac{L}{4\pi d^2} \Rightarrow \frac{L_B}{L_A} = \frac{4\pi d_B^2 B_B}{4\pi d_A^2 B_A} \]

\[ \frac{L_B}{L_A} = \frac{(9.0 \text{ ly})^2 (3.0 \, \mu W/m^2)}{(4.0 \text{ ly})^2 (1.0 \, \mu W/m^2)} = 15 \]
Spectroscopic Parallax

• If luminosity and apparent brightness are known, distance can be determined.

\[ m - M = 5 \log d - 5 \]

• It can be difficult to accurately measure luminosity for one star
  – Use spectra to get spectral type and class

• But, you can use a cluster of stars

• Distance to the cluster can be determined by comparing the HR diagram of the cluster with a template HR diagram
Spectroscopic Parallax

Observations

Apparent Brightness (arbitrary units)

hot O B A F G K M cooler

Template

Luminosity (Solar units)

hot O B A F G K M cooler
Now the vertical axis is on an absolute scale. We can read off the luminosities of the stars, and from that, together with the apparent brightness, we can determine the distance to the stars.
The Distance Ladder

- See Extending Our Reach on p. 484
- First rung: parallax
- Second rung: spectroscopic parallax
- Third rung: standard candles/inverse square law
We will be discussing the longer distance “rungs” of the ladder in subsequent lectures.

Figure 26.12, Freedman and Kaufmann, 7th ed. Universe, © 2005 W. H. Freeman & Company