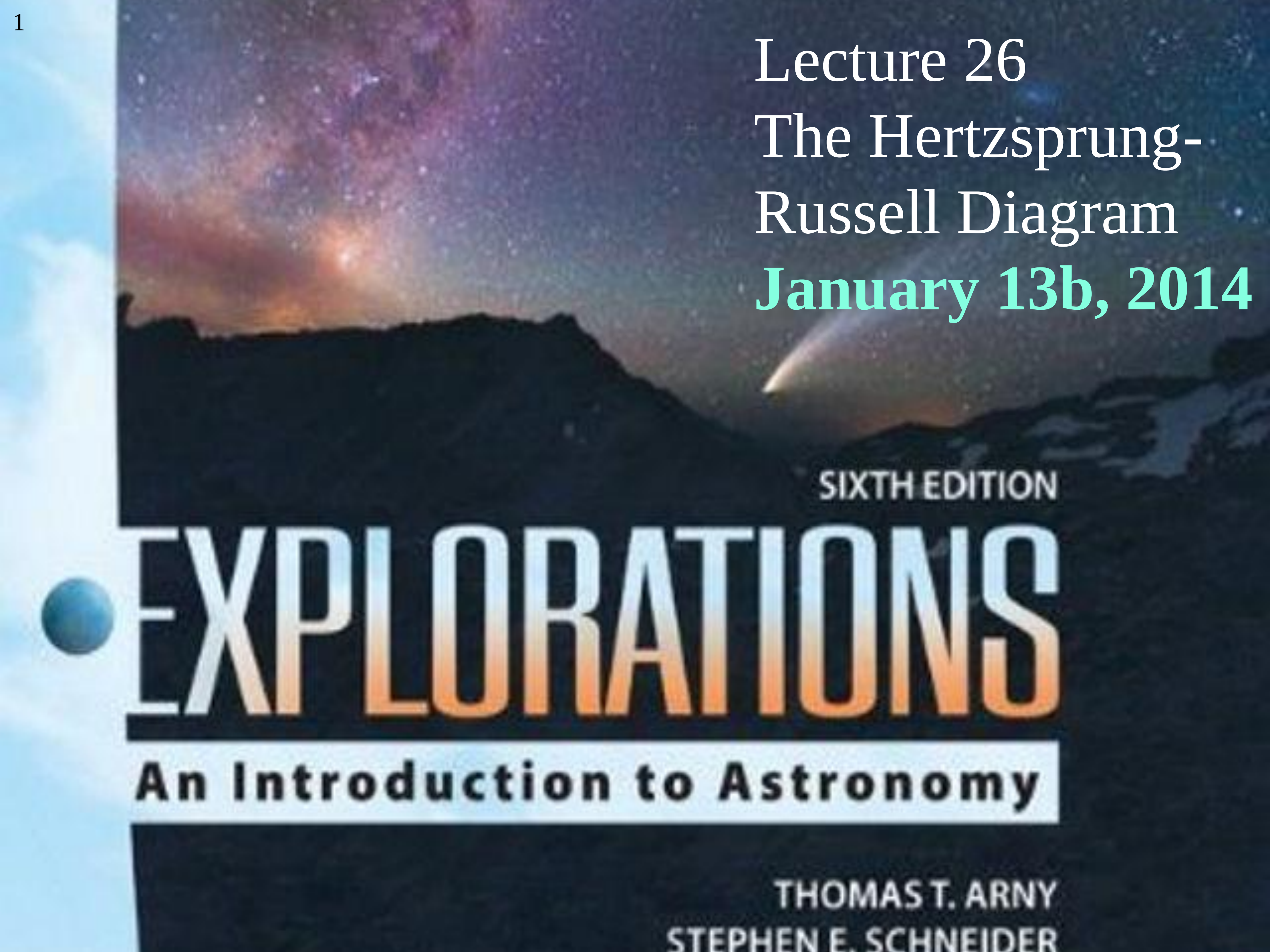
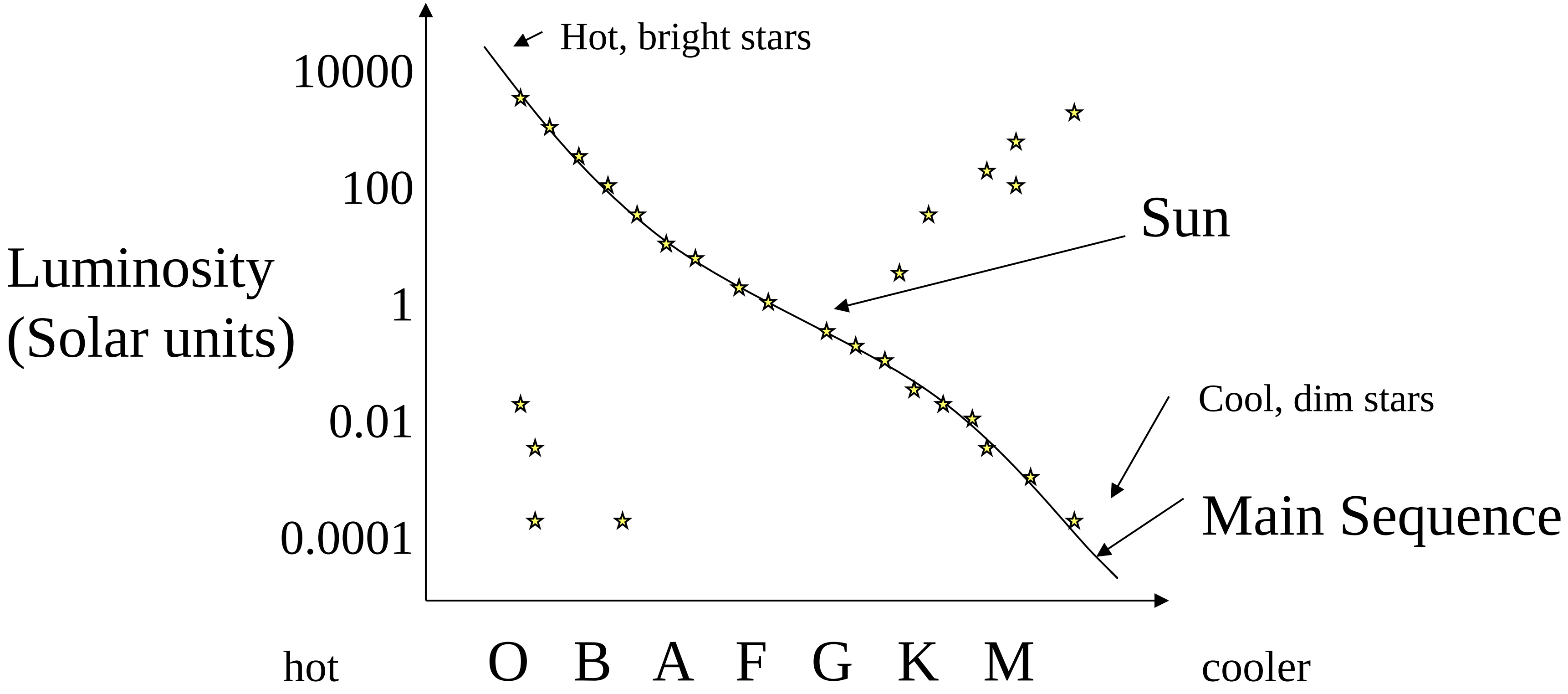
Lecture 26
The Hertzsprung-Russell Diagram

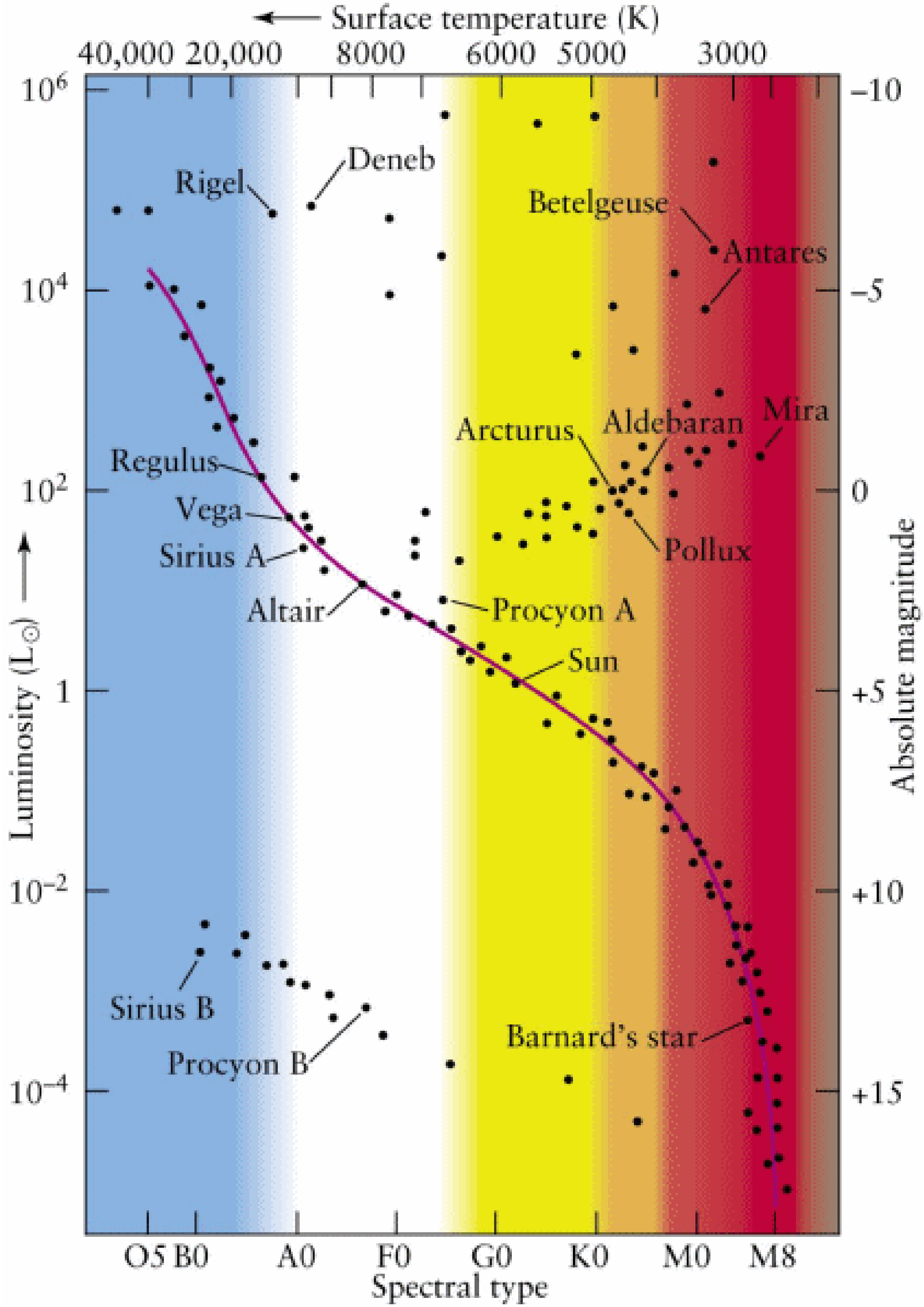
January 13b, 2014



Hertzsprung-Russell Diagram

- Hertzsprung and Russell found a correlation between luminosity and spectral type (temperature)





Surface temperature (K)
40,000 20,000 8000 6000 5000 3000
10^6
10^4
10^2
1
10^{-2}
10^{-4}
Luminosity ($L_\odot$)
−10
−5
0
+5
+10
+15
Absolute magnitude
Rigel
Deneb
Betelgeuse
Antares
Mira
Arcturus
Aldebaran
Regulus
Vega
Sirius A
Pollux
Altair
Procyon A
Sun
Sirius B
Procyon B
Barnard's star
O5 B0 A0 F0 G0 K0 M0 M8
Spectral type

- Most stars (~90%) are on the main sequence
  - The greater the temperature, the more luminous the star
  - M type stars are the most common.
  - O type stars are the least common.

Sizes of Stars

- Size of star is related to the temperature and the luminosity by:

$$\text{Luminosity} \propto \text{Temperature}^4 \times \text{Radius}^2$$

$$L = \sigma T^4 \times 4\pi R^2$$

  - If you know the position on HR diagram you know the size of the star.

A star is twice as luminous as the Sun and has twice the surface temperature. What is its radius relative to the Sun's radius?

A. $R_{\text{star}}/R_{\text{Sun}} = 4.0$

B. $R_{\text{star}}/R_{\text{Sun}} = 2.8$

C. $R_{\text{star}}/R_{\text{Sun}} = 0.5$

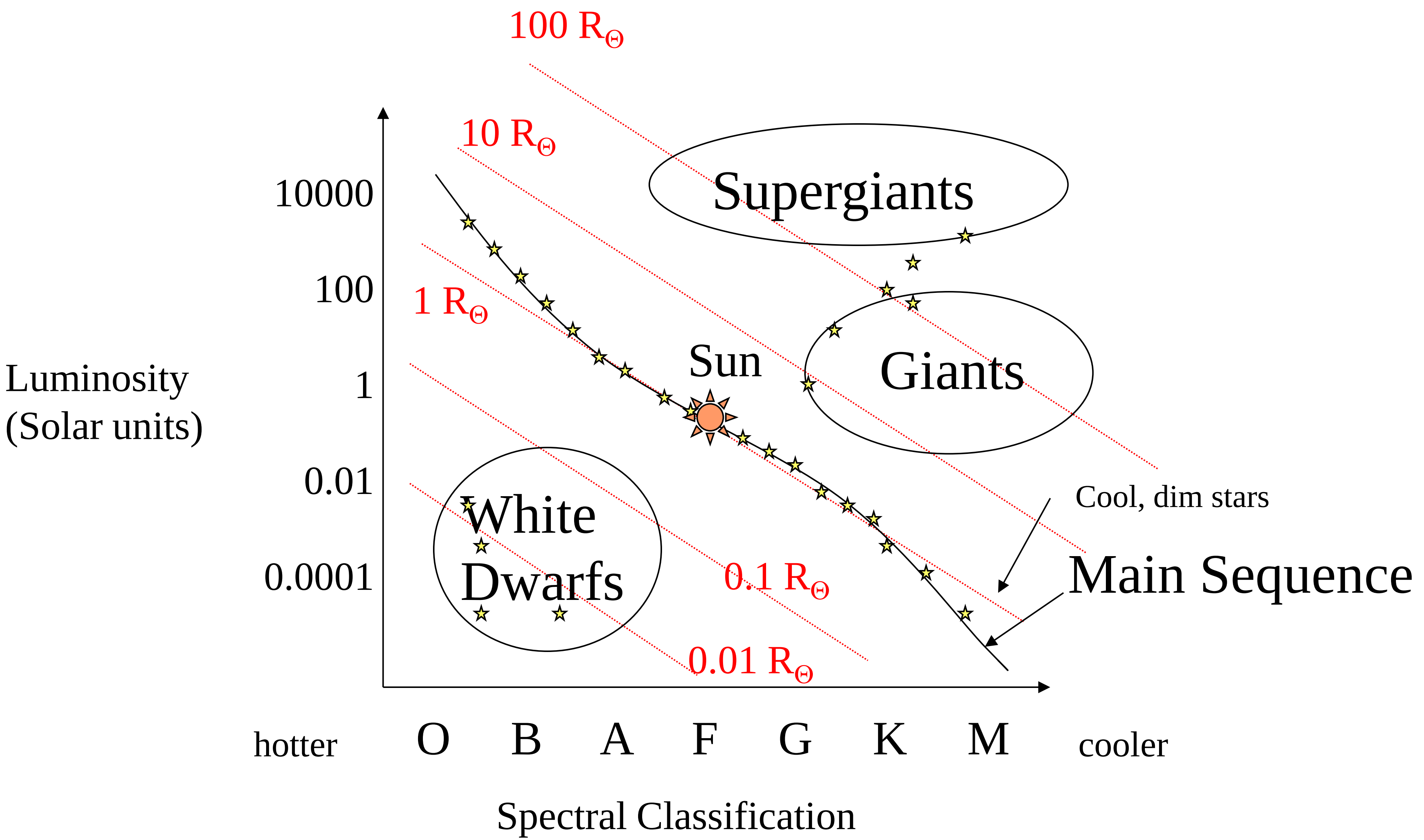
D. $R_{\text{star}}/R_{\text{Sun}} = 0.35$

A star is twice as luminous as the Sun and has twice the surface temperature. What is its radius relative to the Sun's radius?

$$L = \sigma T^4 \, 4\pi R^2 \quad \Rightarrow \quad R = \sqrt{L/4\pi\sigma T^4}$$

$$\frac{R_{\text{star}}}{R_{\text{Sun}}} = \frac{\sqrt{L_{\text{star}}/4\pi\sigma T_{\text{star}}^4}}{\sqrt{L_{\text{Sun}}/4\pi\sigma T_{\text{Sun}}^4}} = \sqrt{\frac{L_{star}}{L_{Sun}}\left(\frac{T_{\text{Sun}}}{T_{\text{star}}}\right)^4}$$

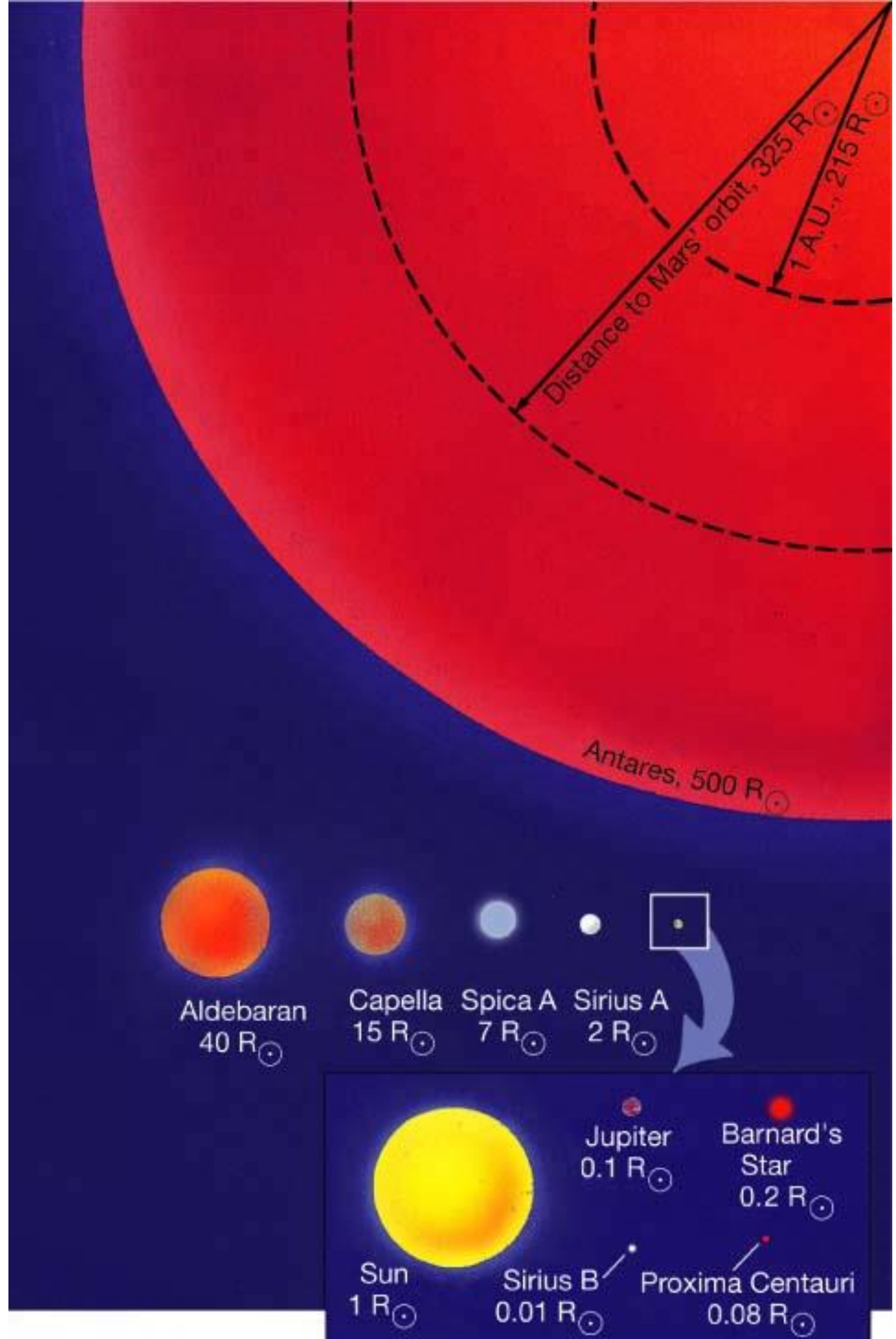
$$= \sqrt{\frac{2}{1}\left(\frac{1}{2}\right)^4} = \boxed{0.35}$$


100 R$_\Theta$
10 R$_\Theta$
1 R$_\Theta$
0.1 R$_\Theta$
0.01 R$_\Theta$
10000
100
1
0.01
0.0001
Luminosity
(Solar units)
Supergiants
Giants
Sun
White
Dwarfs
Cool, dim stars
Main Sequence
hotter
O B A F G K M
cooler
Spectral Classification

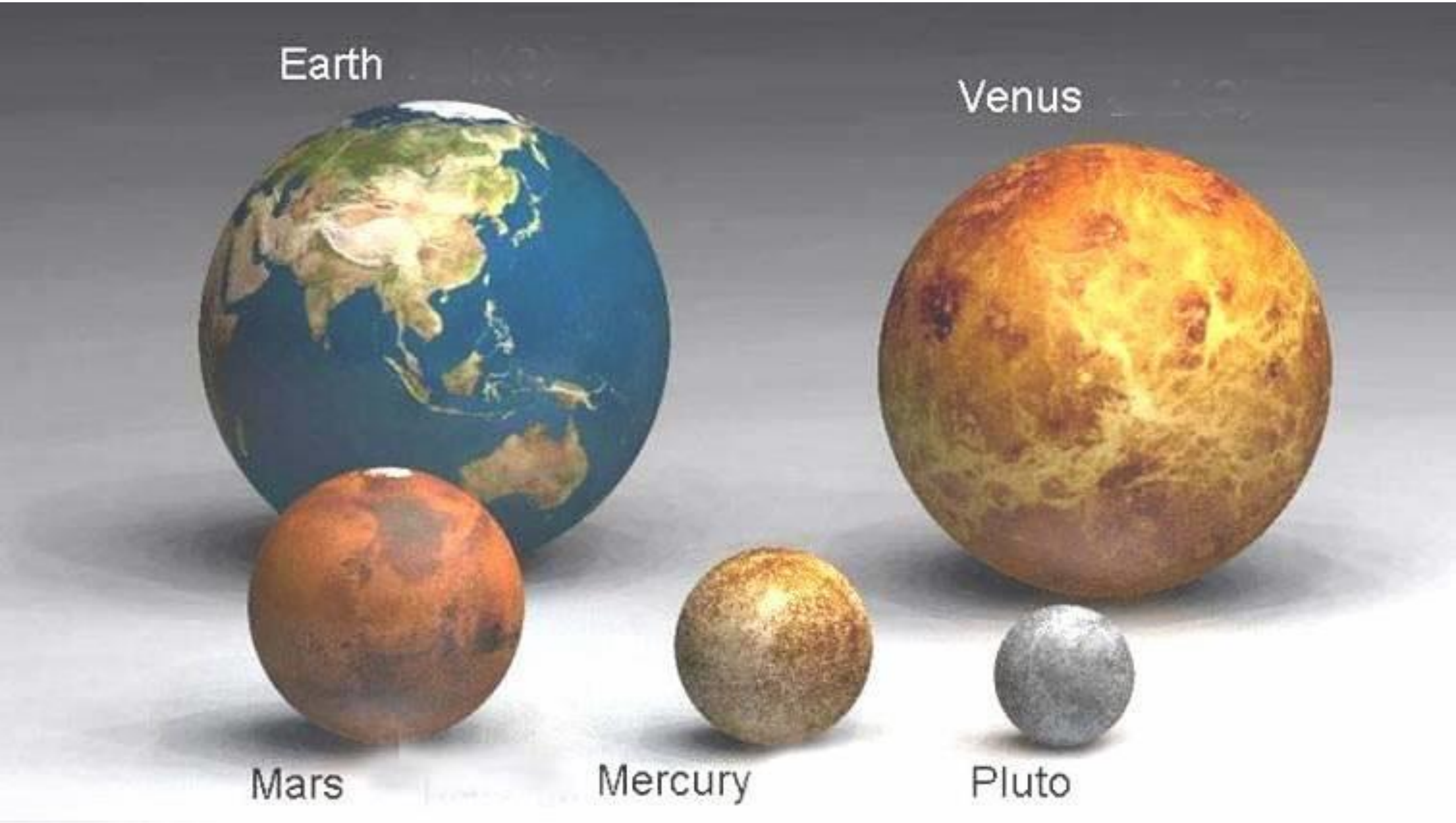
- Supergiants -- cool, bright, red, large stars
- Giants -- cool, bright red, less large stars
- Main Sequence -- spans range from hot, bright stars to cool, dim stars.
- White dwarfs -- hot, small, dim stars.

- These classifications will give clues to stages in the evolution of stars.

Stars come in many sizes!

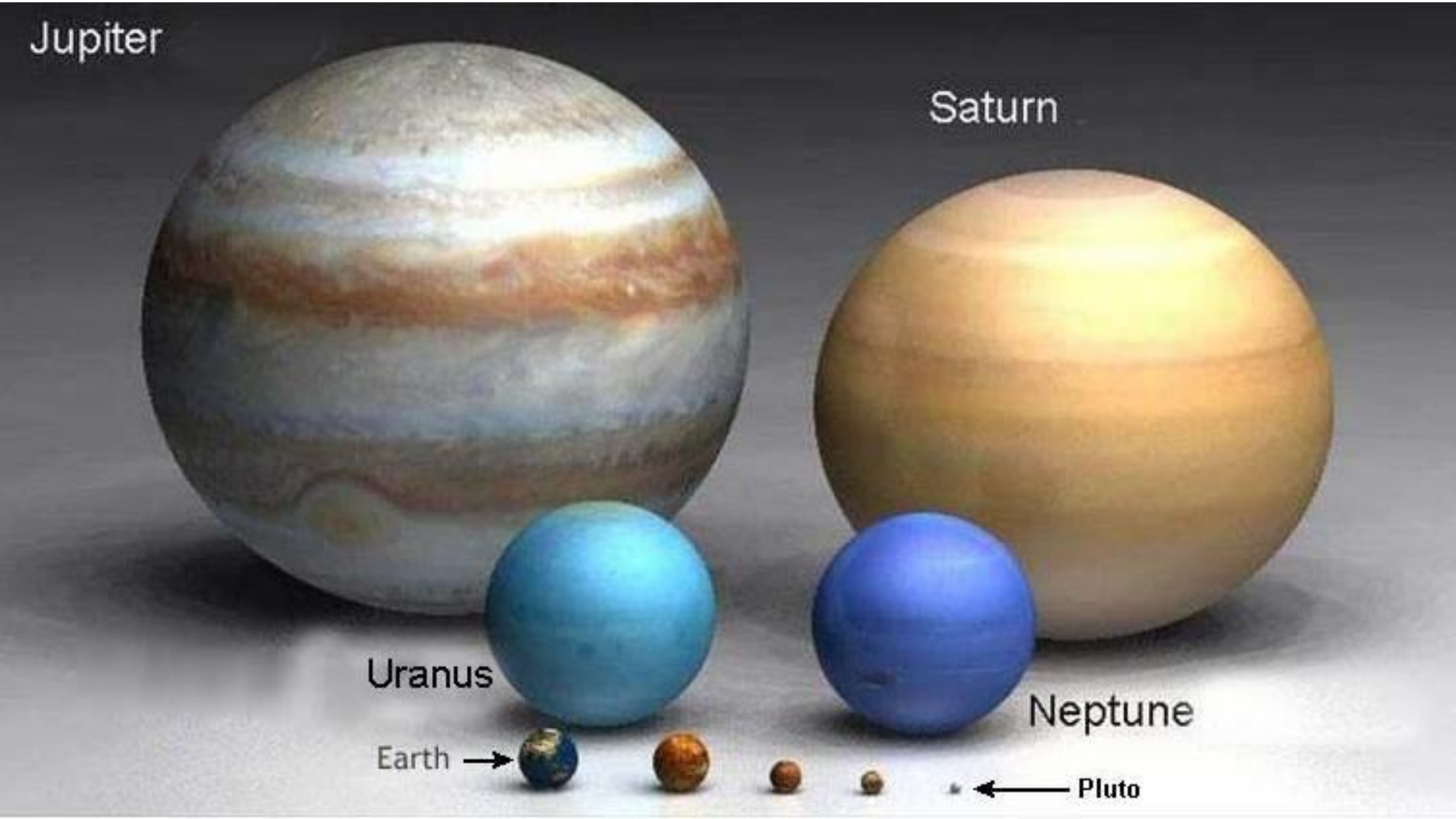


Sizes of Objects in our Universe

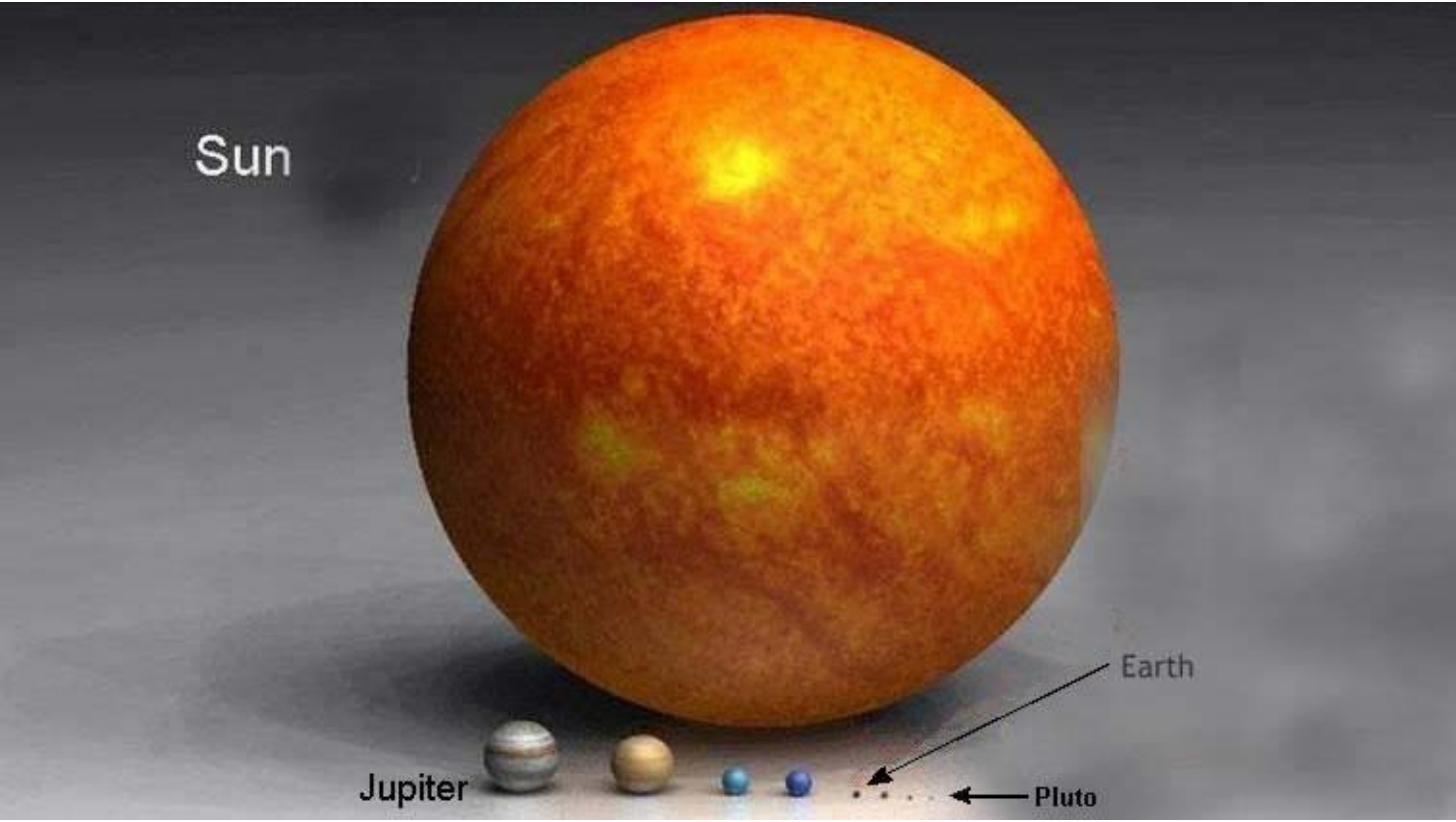


Images from webisto.com/space

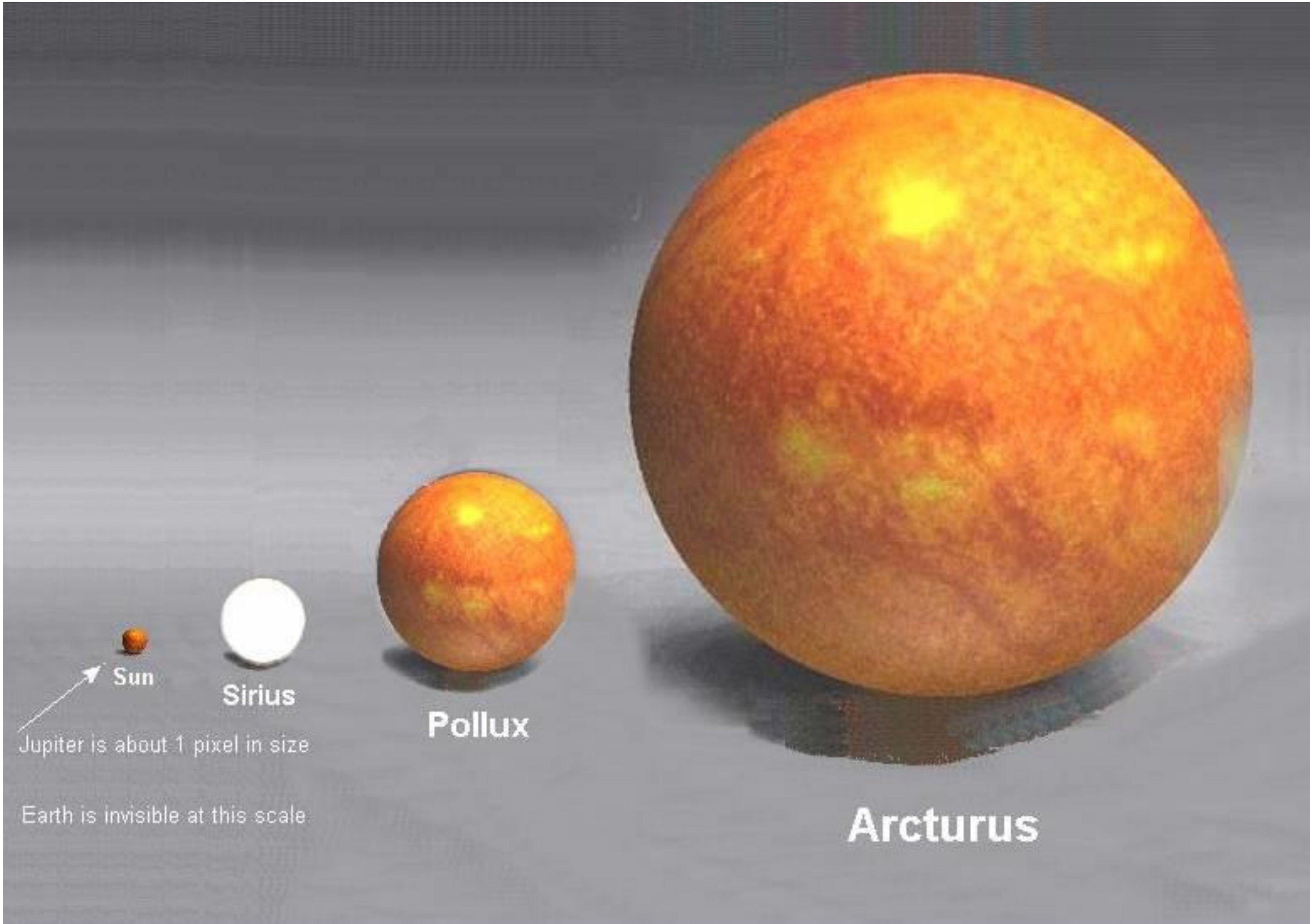
Sizes of Objects in our Universe



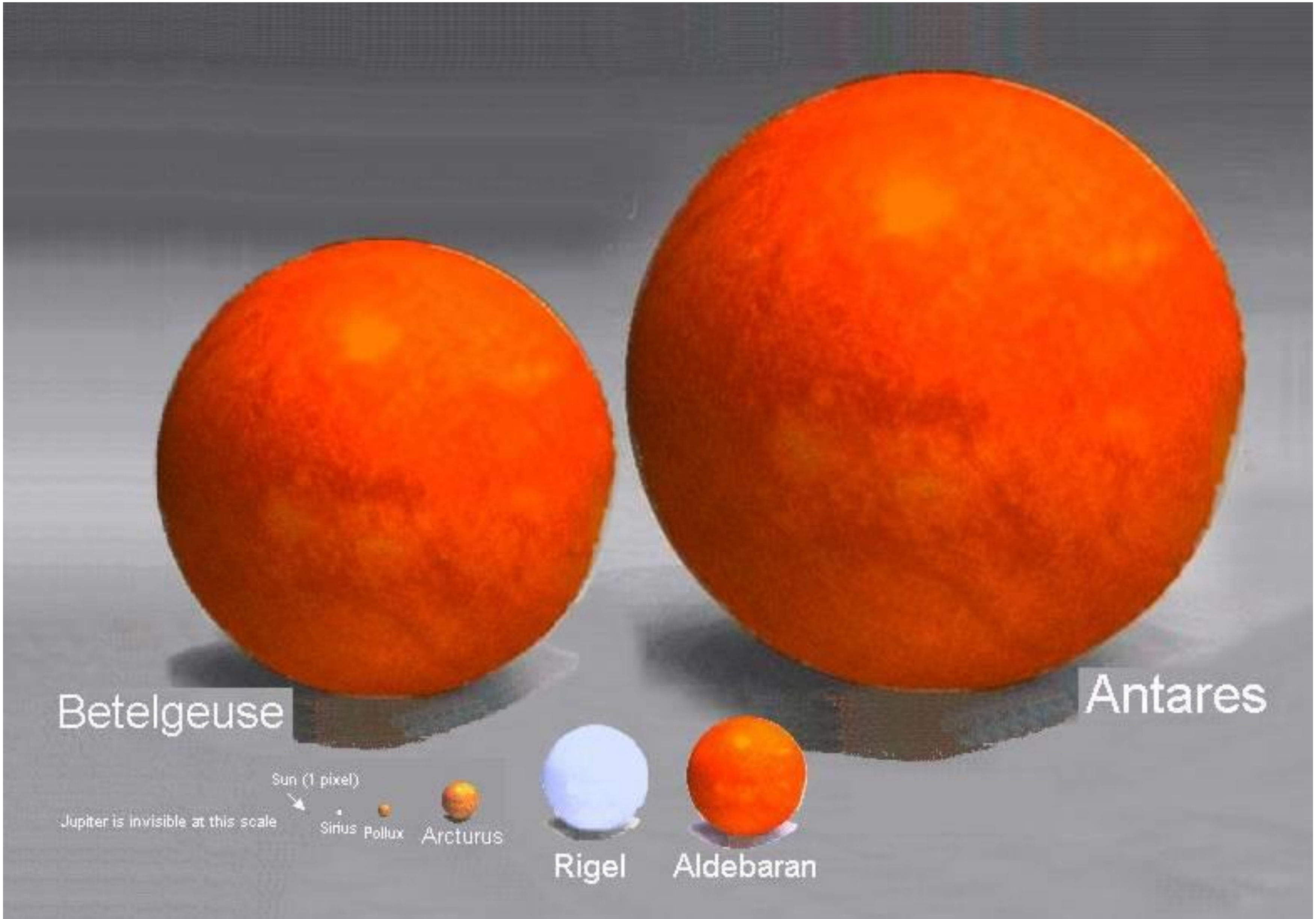
Sizes of Objects in our Universe



Sizes of Objects in our Universe



Sizes of Objects in our Universe



Masses of Stars

- We cannot directly measure the mass of an isolated star.
- If something is orbiting the star, can use the general form of Kepler’s Third Law

$$(M_1 + M_2)P^2 = a^3$$

- Luckily, 2/3 of all stars are binary stars, two stars that orbit one another

A binary star system consists of one star that is twice as massive as the other. They are 2.0 AU apart and have an orbit period of 0.50 y. What is the mass of the smaller star in terms of solar masses?

A. 11 M_{Sun}

B. 4 M_{Sun}

C. 0.50 M_{Sun}

D. 0.125 M_{Sun}

A binary star system consists of one star that is twice as massive as the other. They are 2.0 AU apart and have an orbit period of 0.50 y. What is the mass of the smaller star in terms of solar masses?

A. 11 M_{Sun}

B. 4 M_{Sun}

C. 0.50 M_{Sun}

D. 0.125 M_{Sun}

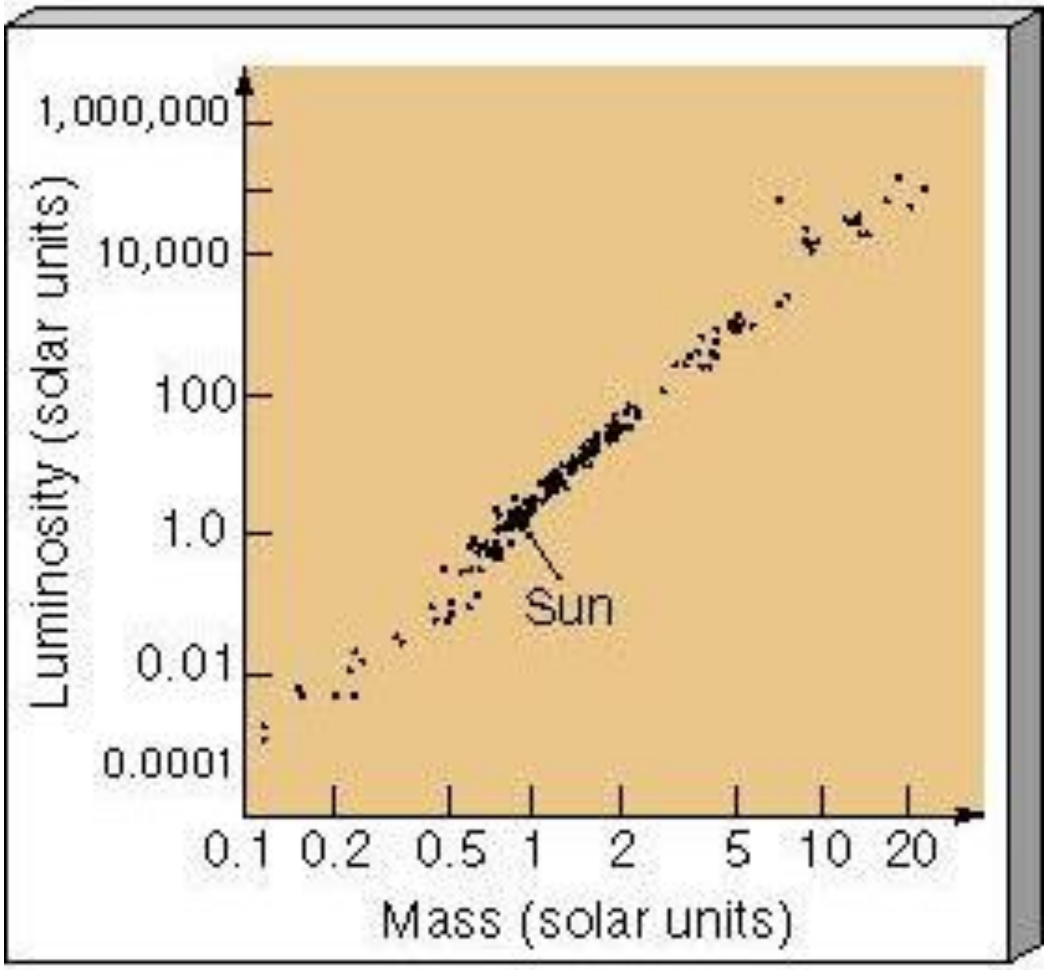
$$(m + 2m)P^2 = a^3$$

$$m = \frac{a^3}{3P^2} = \frac{(2.0\ \mathrm{AU})^3}{3(0.50\ \mathrm{y})^2} = \boxed{11 M_\odot}$$

Mass-Luminosity Relation

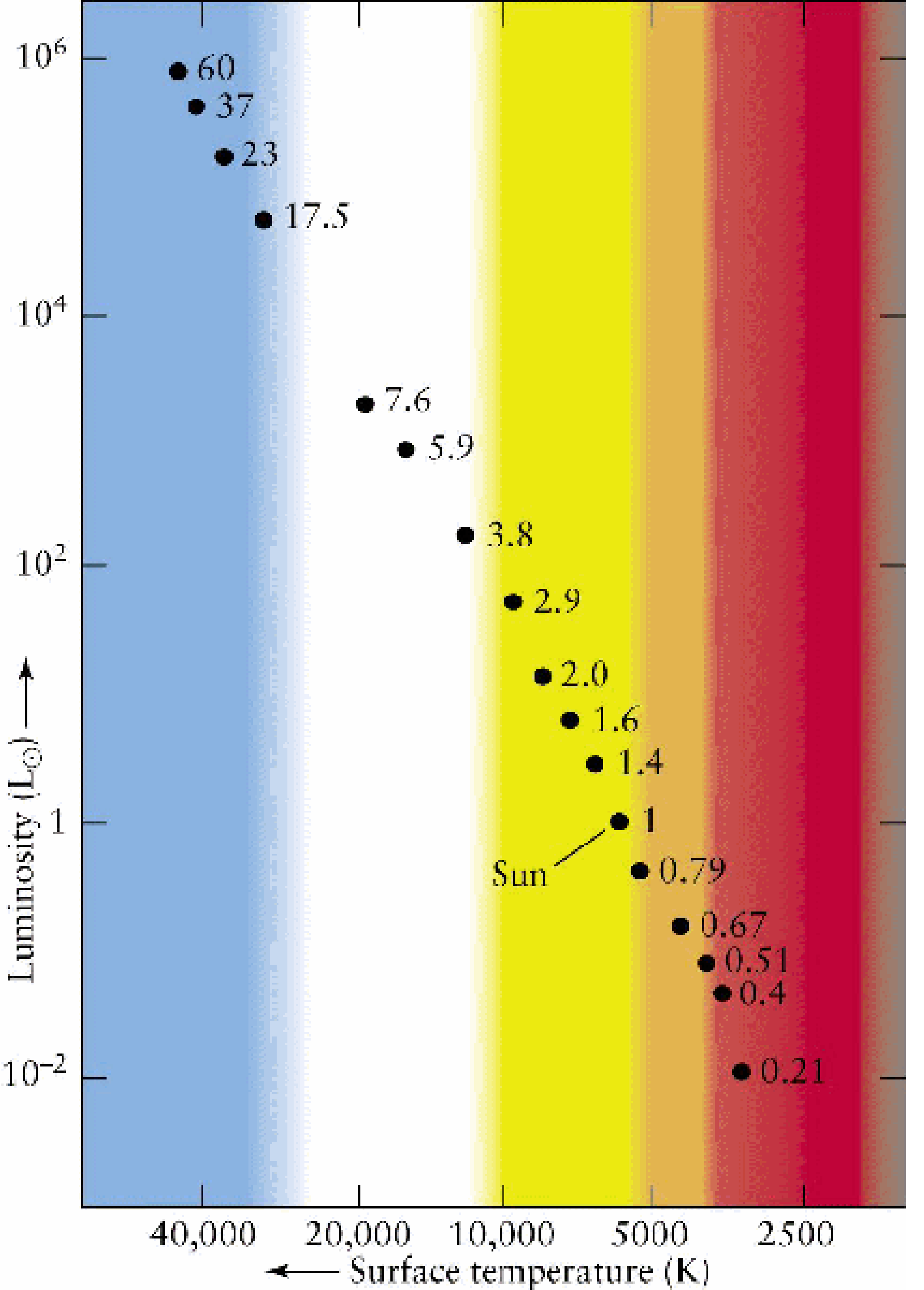
- True ONLY for Main Sequence stars
- As the mass increases, the luminosity increases rapidly

$$\text{Luminosity} \propto \text{Mass}^3$$



(b)

HR Diagram: Main sequence stars labeled by mass in units of solar masses.



Question

How do the following properties differ with respect to the Sun for the listed main sequence types? What properties would not change for *non*-main sequence stars?

	O	G	M
Temperature			
Mass			
Size			
Color			
Luminosity			

Question

How do the following properties differ with respect to the Sun for the listed main sequence types? What properties would not change for *non*-main sequence stars?

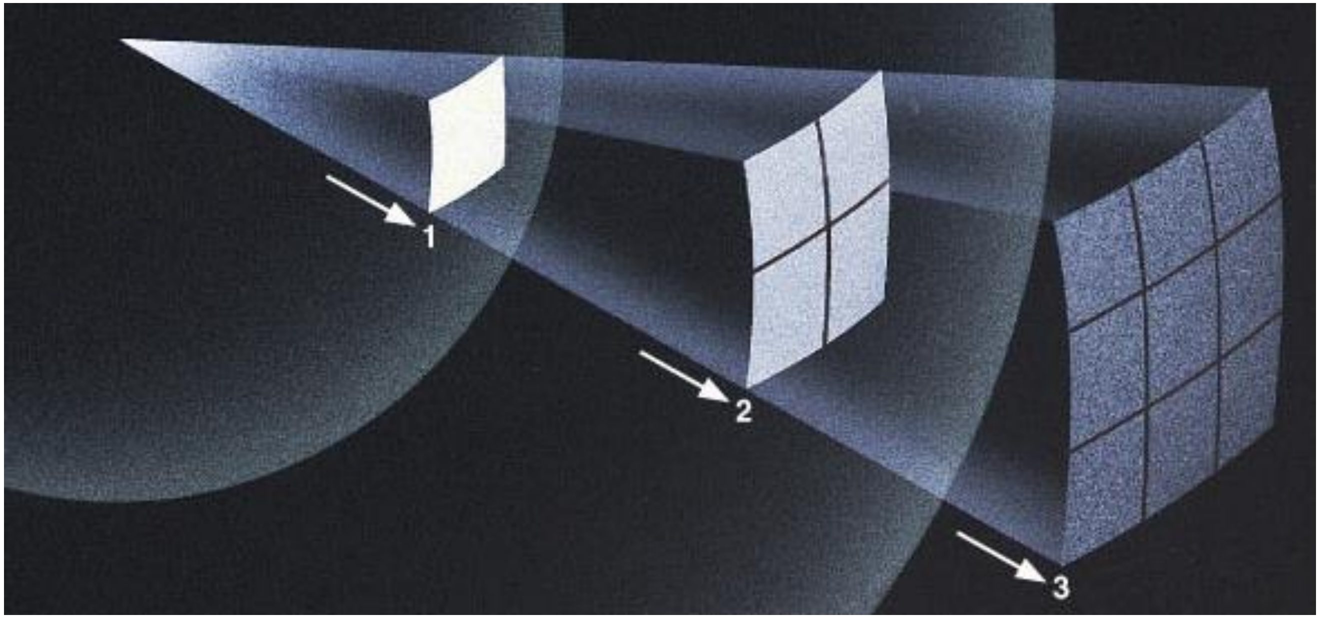
	O	G	M
Temperature*	Higher	Same	Lower
Mass	Higher	Same	Lower
Size	Larger	Same	Smaller
Color*	Blue	White	Red
Luminosity	Higher	Same	Red

Apparent Brightness

- The brightness an object appears to have.
- The further away the object, the dimmer it looks

$$\text{Apparent Brightness} = \frac{\text{Luminosity}}{4\pi d^2}$$

d = distance



Star A has a brightness of 1.0 μW/m^2 and is known to be 4.0 ly away. Star B is 9.0 ly away and has a brightness of 3.0 μW/m^2. How much more luminous is star B than star A?

A. 240×

B. 15×

C. 6.8×

D. 3.0×

Star A has a brightness of 1.0 μW/m^2 and is known to be 4.0 ly away. Star B is 9.0 ly away and has a brightness of 3.0 μW/m^2. How much more luminous is star B than star A?

A. 240×

B. 15×

C. 6.8×

D. 3.0×

$$B = \frac{L}{4\pi d^2} \quad \Rightarrow \quad \frac{L_\mathrm{B}}{L_\mathrm{A}} = \frac{4\pi d_\mathrm{B}^2 B_\mathrm{B}}{4\pi d_\mathrm{A}^2 B_\mathrm{A}}$$

$$\frac{L_\mathrm{B}}{L_\mathrm{A}} = \frac{(9.0\ \mathrm{ly})^2 (3.0\ \mu\mathrm{W/m^2})}{(4.0\ \mathrm{ly})^2 (1.0\ \mu\mathrm{W/m^2})} = \boxed{15}$$

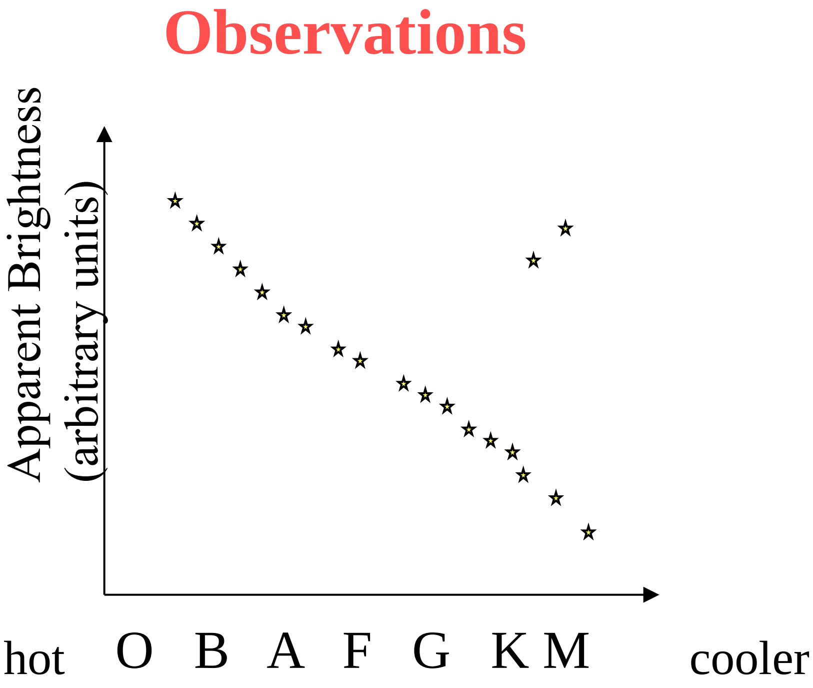
Spectroscopic Parallax

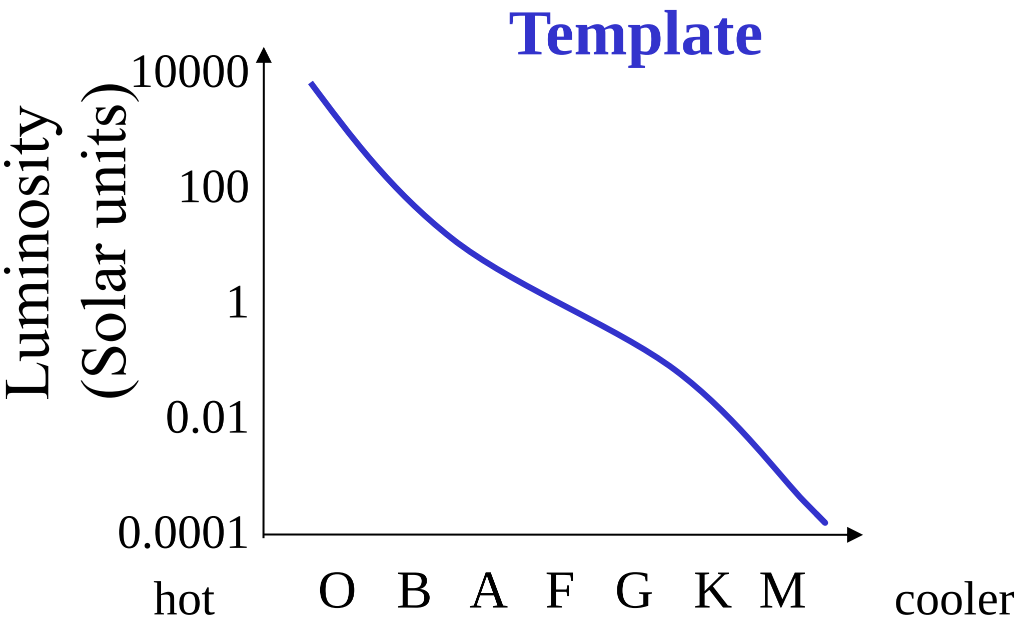
- If luminosity and apparent brightness are known, distance can be determined.

$$m - M = 5\log d - 5$$

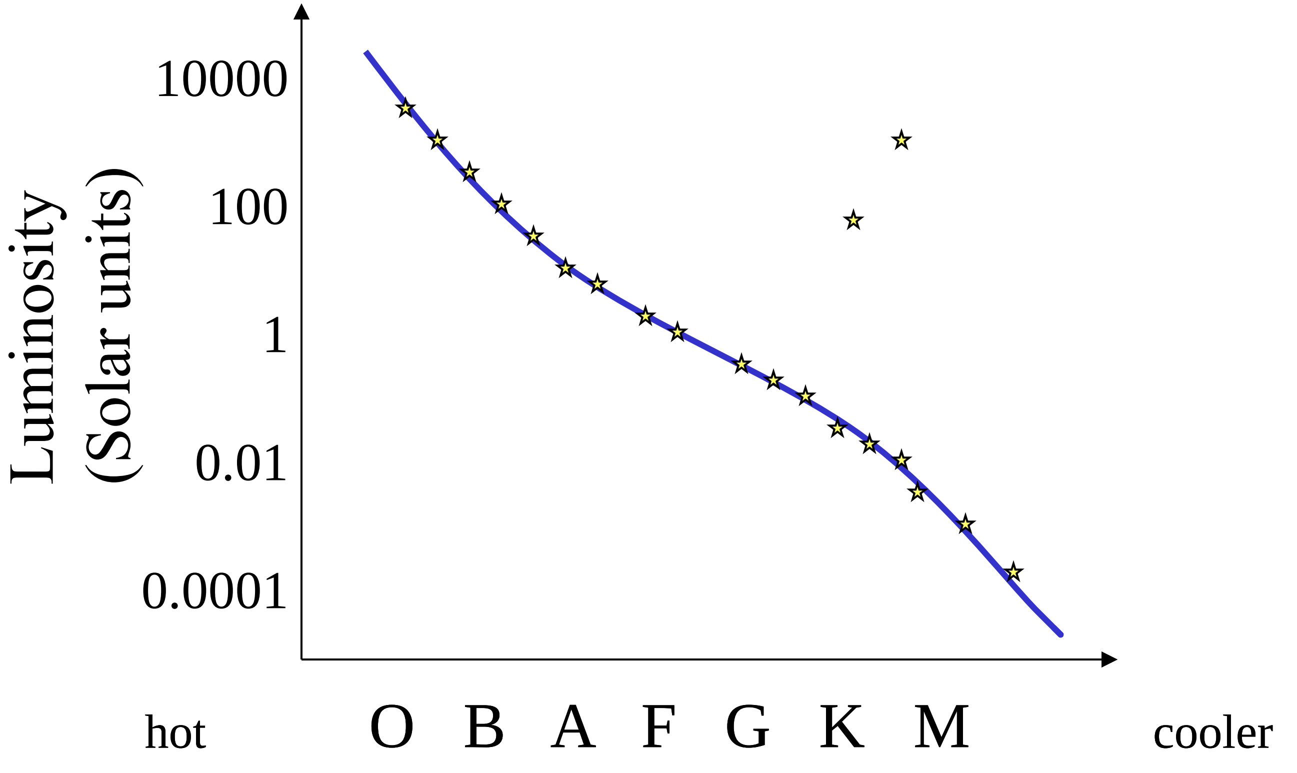
- It can be difficult to accurately measure luminosity for one star
  - Use spectra to get spectral type and class
- But, you can use a *cluster* of stars
- Distance to the cluster can be determined by comparing the HR diagram of the cluster with a template HR diagram

Spectroscopic Parallax





Spectroscopic Parallax

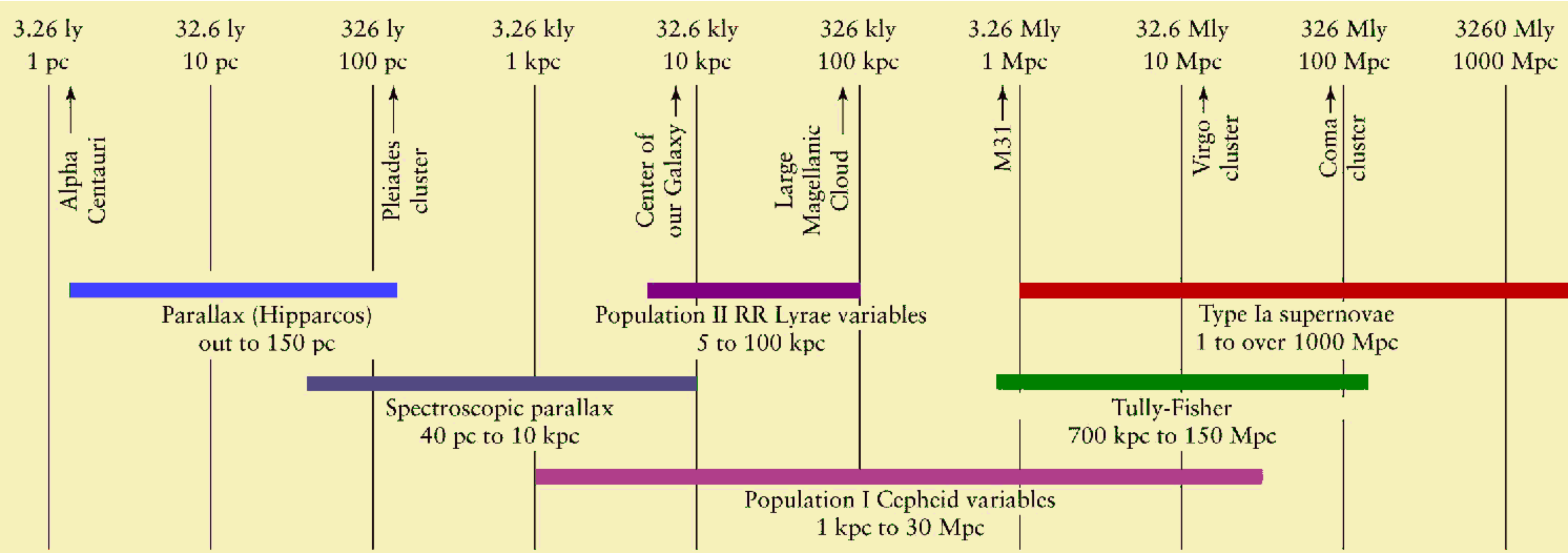


Now the vertical axis is on an absolute scale. We can read off the luminosities of the stars, and from that, together with the apparent brightness, we can determine the distance to the stars.

The Distance Ladder

- See Extending Our Reach on p. 484
- First rung: parallax
- Second rung: spectroscopic parallax
- Third rung: standard candles/inverse square law

The Distance Ladder



We will be discussing the longer distance "rungs" of the ladder in subsequent lectures.

Figure 26.12, Freedman and Kaufmann,
7th ed. *Universe*,