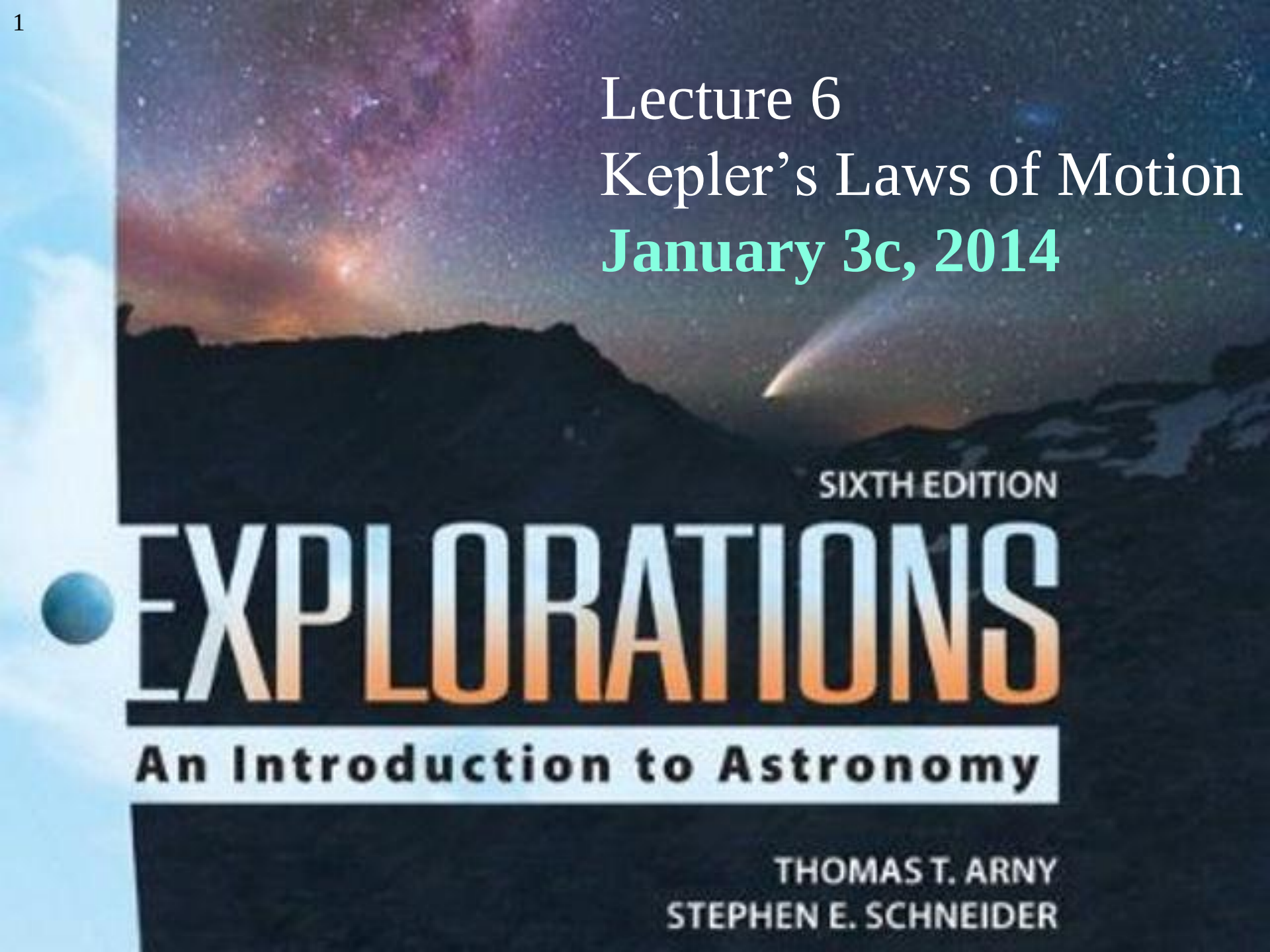


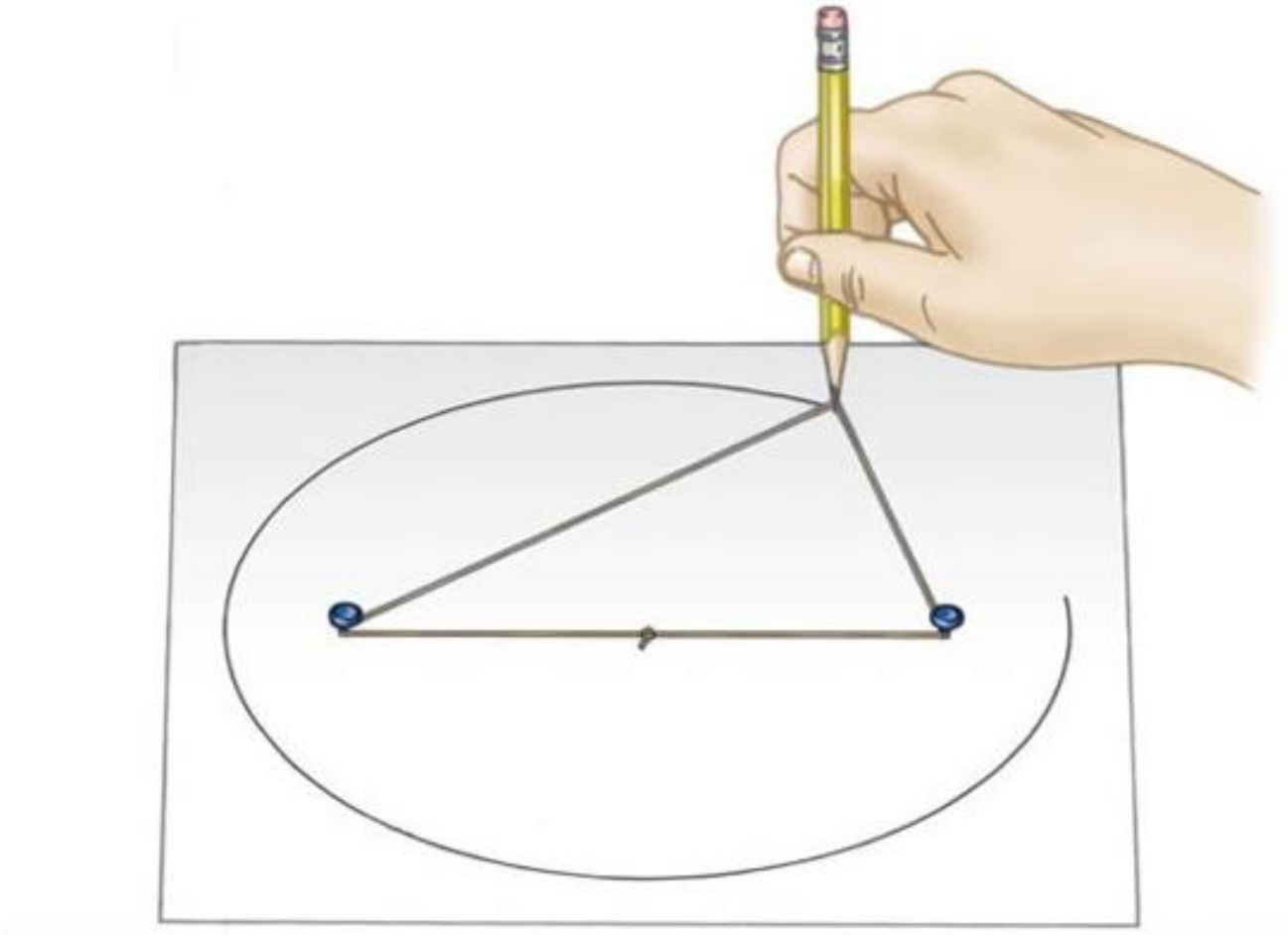
Lecture 6
Kepler's Laws of Motion
January 3c, 2014

Johannes Kepler (1571-1630)

- Assistant to Brahe (1600-01)
- Believed in heliocentric model (Copernicus).
- Wanted mathematical model of orbits
- Used observations by Brahe
- Used trial and error to test models

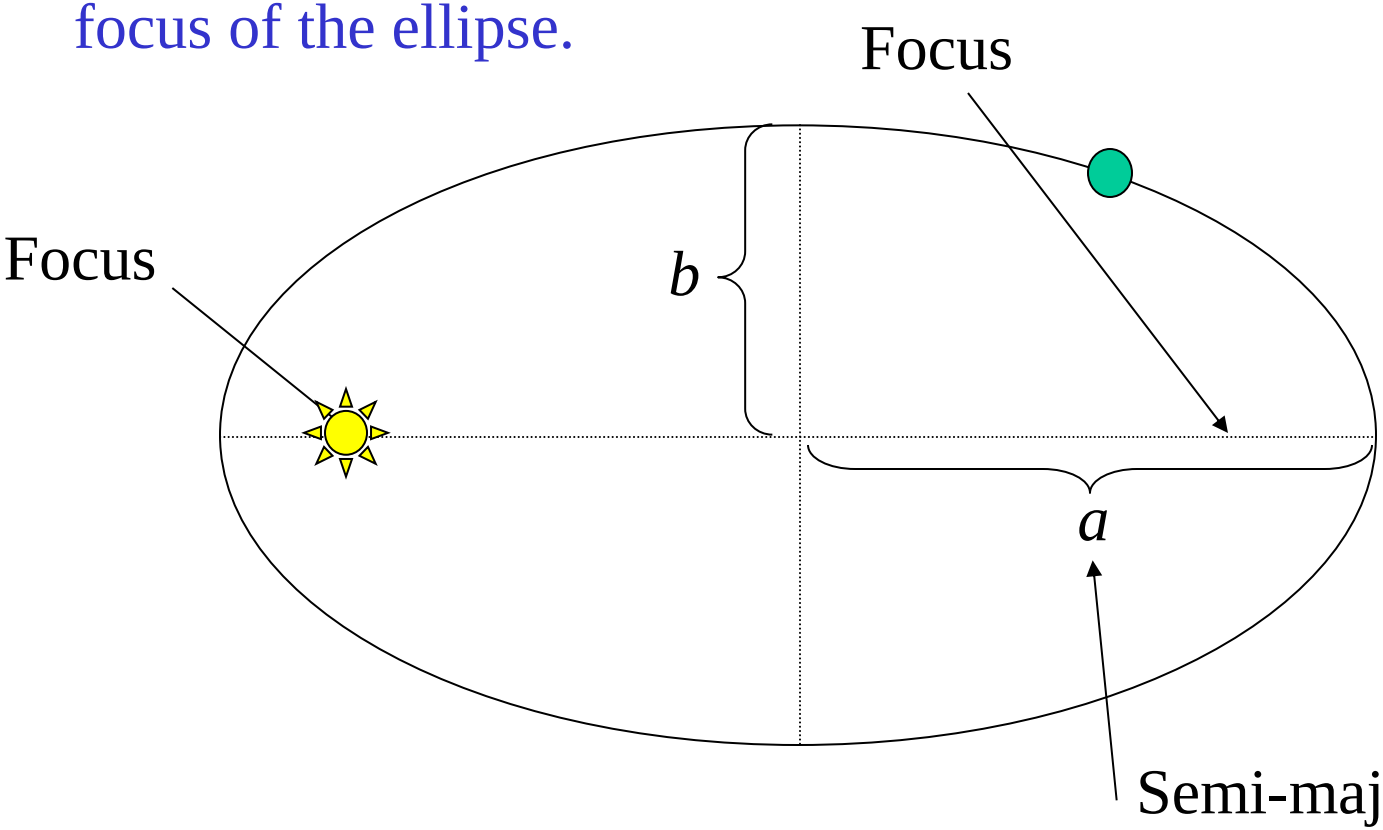
Kepler's First Law

- The orbit of a planet about the Sun is an ellipse with the Sun at one focus.



Elliptical Orbit

There is nothing physically at the second focus of the ellipse.



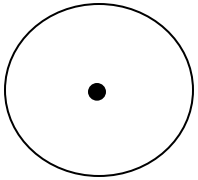
Animation applet

For the Sun and the planets the orbits are almost circular.

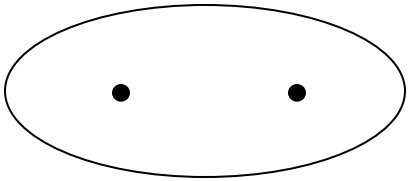
Eccentricity = Shape of Orbit

- Values range from 0 to 1

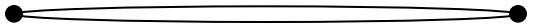
→ 0 = perfect circle



→ 0.5 = ellipse



→ 1.0 = straight line

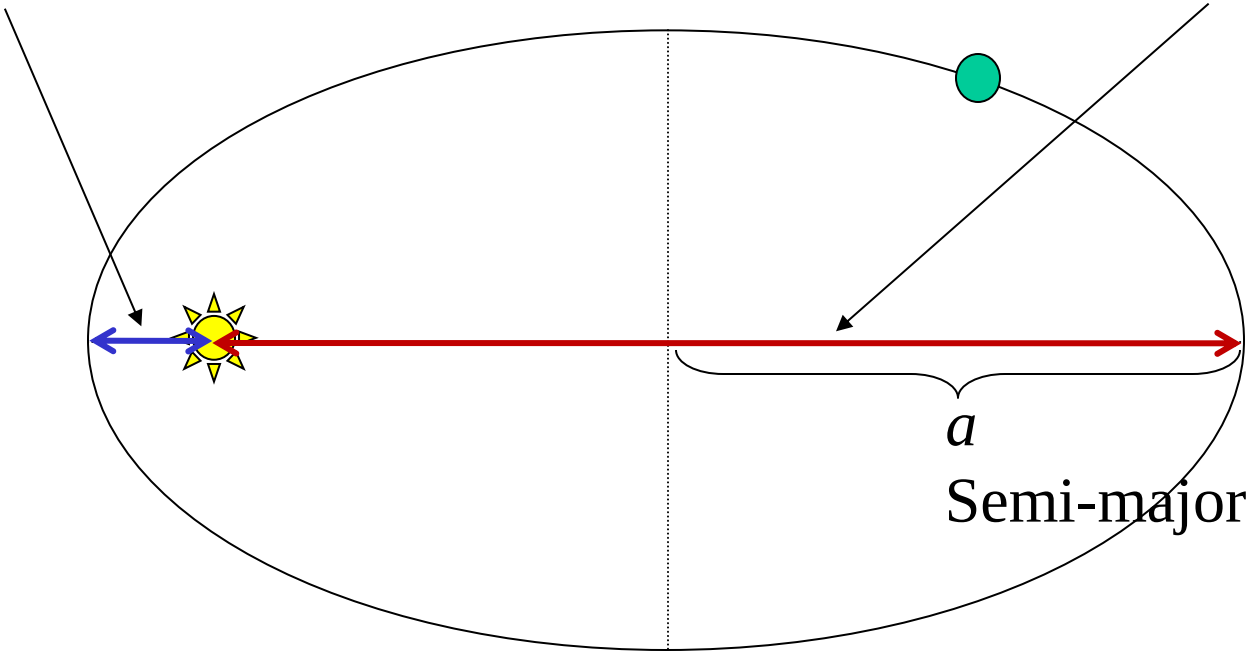


$$\varepsilon = \sqrt{1 - \frac{b^2}{a^2}}$$

Elliptical Distances

Perihelion: closest distance to the Sun

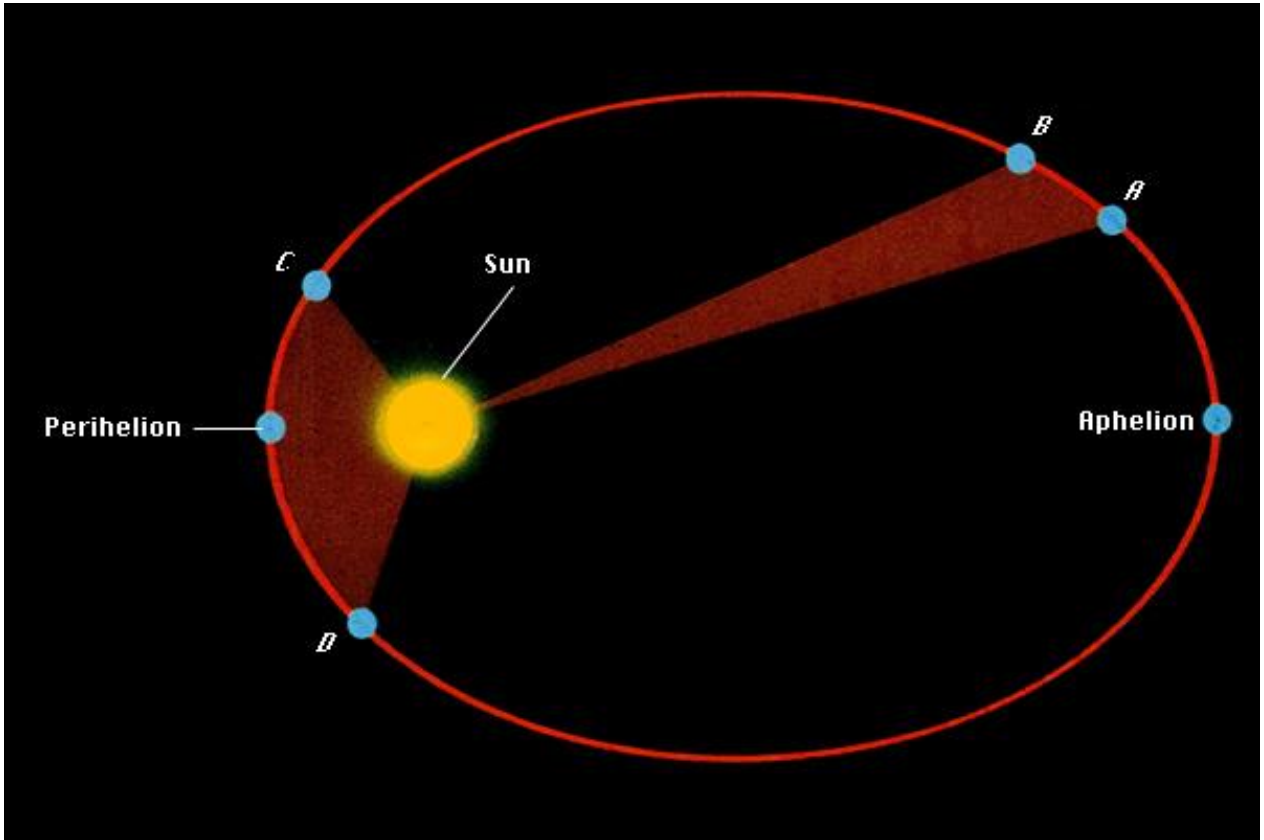
Aphelion: furthest distance from the Sun



Note that $r_{\text{perihelion}} + r_{\text{aphelion}} = 2a$

Kepler's Second Law

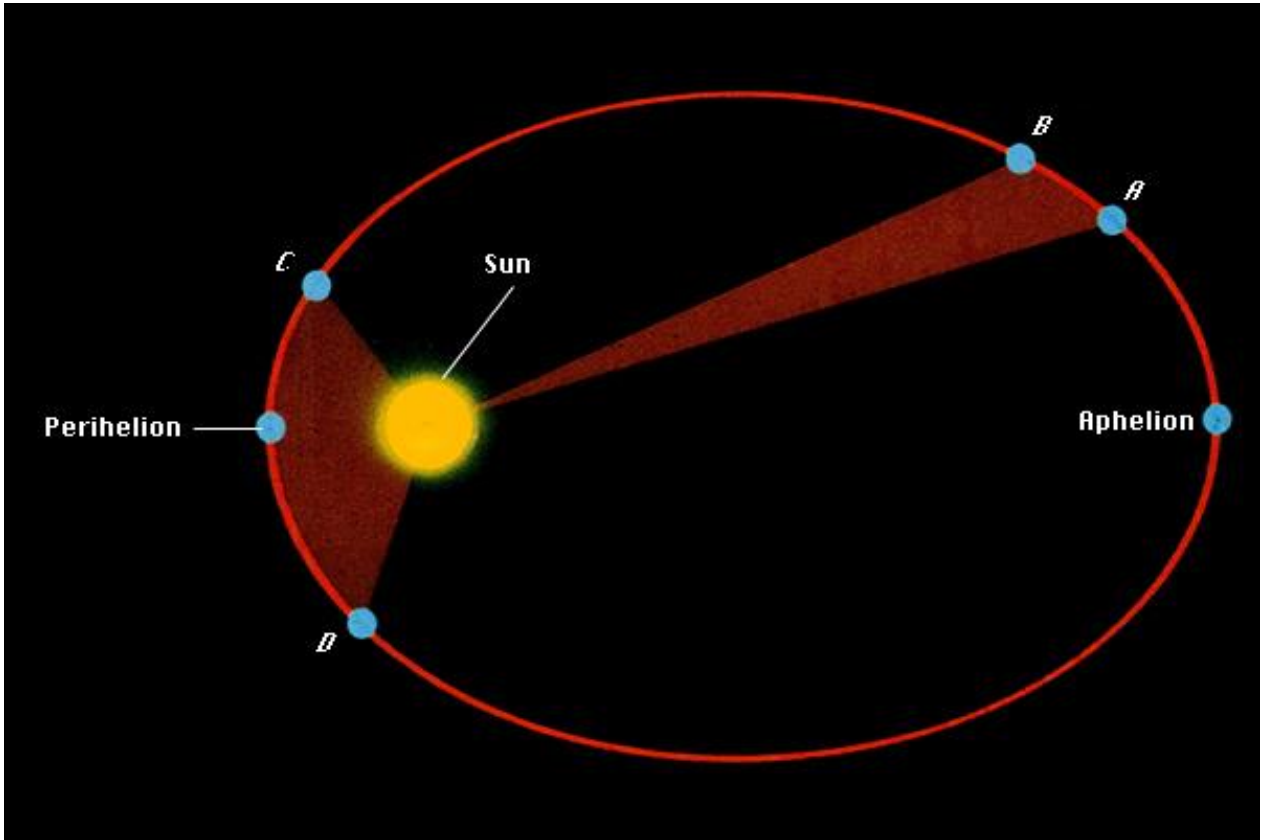
- "The Law of Equal Areas" -- A line joining a planet and the Sun always sweeps out equal areas in equal intervals of time.



Animation applet

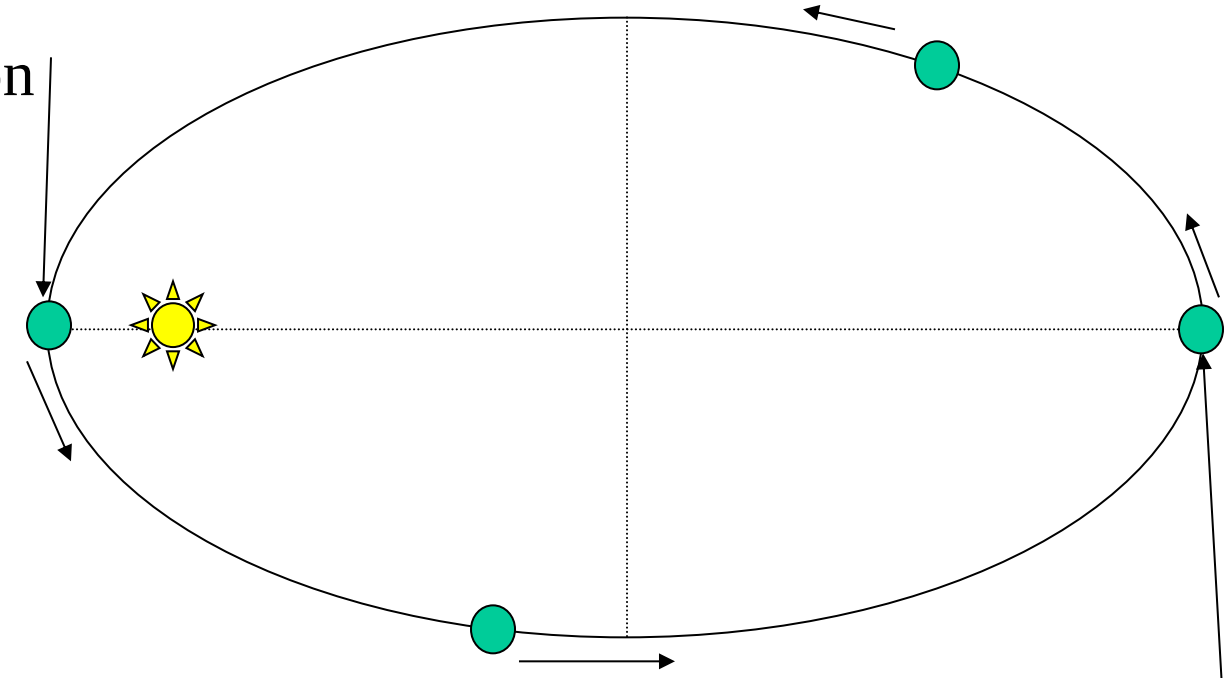
Question

- Does the speed of an object change in an elliptical orbit? If so, where does it move fastest and where does it move slowest?



Speed of Planets in Elliptical Orbits

Planet moves fastest at perihelion



Planet moves slowest at aphelion

Animation

Kepler's Third Law

- $P^2 = a^3$
  - P = sidereal period in years
  - a = semi-major axis in AU
    - AU = Astronomical Unit = Average distance between the Earth and the Sun
  - The closer a planet is to the Sun, the less time it takes to go around the Sun.

Example: If $P = 8$ years then

$$P^2 = 64 = a^3$$

$$\Rightarrow a = \sqrt[3]{64} = 4 \text{ AU}$$

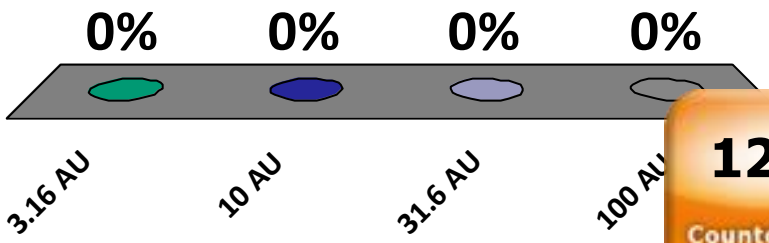
A new comet is detected with a period of 1000 years. What is its semi-major axis?

A. 3.16 AU

B. 10 AU

C. 31.6 AU

D. 100 AU



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Countdown Timer On Slide

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A comet is detected with a period of 1000 years. What is its semi-major axis?

A. 100 AU

B. 31.6 AU

C. 10 AU

D. 3.16 AU

If $P = 1000$ years then

$$P^2 = 1{,}000{,}000 = a^3$$

$$\Rightarrow a = \sqrt[3]{1{,}000{,}000} = \boxed{100 \text{ AU}}$$

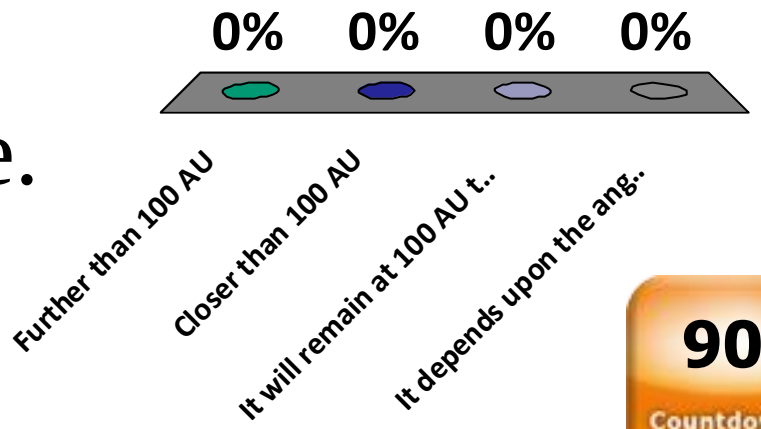
A new comet is detected with a period of 1000 years. If its eccentricity is high (0.9), will it spend more time further than 100 AU or closer than 100 AU from the Sun?

A. Further than 100 AU

B. Closer than 100 AU

C. It will remain at 100 AU the entire time

D. It depends upon the angle of its orbital plane.



90
Countdown Timer On Slide

0 of 5

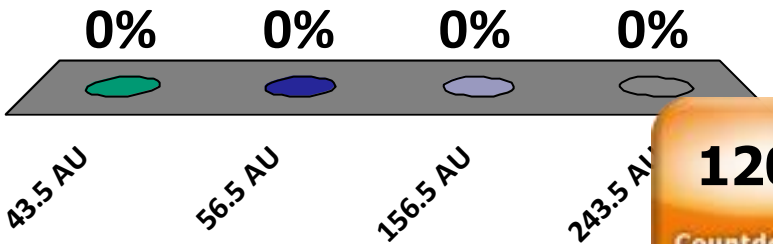
A new comet is detected with a period of 1000 years. If its perihelion distance is 43.5 AU, what is its aphelion distance?

A. 43.5 AU

B. 56.5 AU

C. 156.5 AU

D. 243.5 AU



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Countdown Timer On Slide

0 of 5

A new comet is detected with a period of 1000 years. If its perihelion distance is 43.5 AU, what is its aphelion distance?

A. 43.5 AU

B. 56.5 AU

C. 156.5 AU

D. 243.5 AU

$$r_{\text{perihelion}} + r_{\text{aphelion}} = 2a = 200 \text{ AU}$$

$$r_{\text{aphelion}} = 200 \text{ AU} - 43.5 \text{ AU} = \boxed{156.5 \text{ AU}}$$

General Comments on Kepler's Laws

- They apply to all orbiting bodies (moons, planets, stars, galaxies..), although the mathematics of the 3rd law need slight modification.
- Kepler's model was constructed to fit the data – it was NOT derived from physical principles
- It is simpler than the Copernican model, and fits the data more accurately