What is Formal (or Symbolic) Logic?

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You’ll Learn

Concepts

• What arguments (including conclusions, premises, and inferences) are.
• What validity is.
• Some of the differences between Informal Logic (Critical Thinking) and Formal Logic (Symbolic Logic).

Skills

• How to symbolize some arguments.
• How to demonstrate that arguments are valid by constructing proofs using Modus Ponens and Modus Tollens.
• How to demonstrate that arguments are invalid by identifying the Fallacy of Assuming the Consequent and the Fallacy of Negating the Antecedent.
Some Concepts from Formal Logic

I. "If Alma is invited then the party is certain to be a success. Alma is invited. Therefore the party is certain to be a success."

• **Argument** (a unit of reasoning that attempts to prove that a certain idea is true by citing other ideas as evidence)
• **Conclusion** (the idea that the argument is trying to prove true)
• **Premises** (the ideas that argument is assuming to be true and advancing as evidence for the ultimate conclusion)
• **Inference** (the connection that holds between the premises and the conclusion; the extent to which the truth of the premises would establish the truth of the conclusion.)
I. “If Alma is invited then the party is certain to be a success. Alma is invited. Therefore the party is certain to be a success.”

- The premises of this argument may or may not be true.
- If the premises of this argument are true then the conclusion of this argument must be true.
- The inference in this argument is perfect – or valid.

Informal Logic (Critical Thinking)
- is concerned with evaluating premises and inferences,
- evaluates inferences on a sliding scale,
- considers arguments in the “natural language”
I. “If Alma is invited then the party is certain to be a success. Alma is invited. Therefore the party is certain to be a success.”

Formal Logic (Symbolic Logic)
- doesn’t evaluate premises, focusing entirely on inferences,
- evaluates inferences as “valid” or “invalid,”
- translates arguments into a formal language.

First Commandment of Formal Logic:
Thou shall relax in the presence of Symbols, for they art thy Friends.

Some Skills from Formal Logic

Symbolizing Arguments
and
Constructing Proofs
*Modus Ponens*
I. “If Alma is invited then the party is certain to be a success. Alma is invited. Therefore the party is certain to be a success.”

“If A then S. A. Therefore S.”

\[ A \rightarrow S, A \vdash S \]

*Modus Ponens – Valid Inference Form*

\[ P \rightarrow Q, P \vdash Q \]

“If Max is poodle then Max is a dog. Max is a poodle. Therefore Max is a dog.”

Proof (a demonstration of how we can work our way from the premises to the conclusion of the argument using only valid inference forms)

\[
\begin{align*}
1. & \ A \rightarrow S \\
2. & \ A \\
3. & \ S \\
\end{align*}
\]

Proof (a demonstration of how we can work our way from the premises to the conclusion of the argument using only valid inference forms)

If we can construct a proof for an argument then the argument is valid.
II. “If Alma is invited then the party is certain to be a success. Alma is invited. If the party is certain to be a success then Joan will be delighted. Therefore the Joan will be delighted.”

\[ A \rightarrow S, A, S \rightarrow J \vdash J \]

1. \[ A \rightarrow S \quad P \]
2. \[ A \quad P \]
3. \[ S \rightarrow J \quad P \]
4. \[ S \quad 1, 2 \text{ MP} \]
5. \[ J \quad 3, 4 \text{ MP} \]

This argument is valid.

Some Skills from Formal Logic

Symbolizing Arguments and Constructing Proofs using *Modus Tollens*
III. “If she’s at home then she’ll come to the door. She doesn’t come to the door. Therefore she isn’t at home.”

\[ H \rightarrow D, \sim D \mid \sim H \]

*Modus Tollens – Valid Inference Form*

\[ P \rightarrow Q, \sim Q \mid \sim P \]

“If Max is poodle then Max is a dog. Max is not a dog. Therefore Max is not a poodle.”

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III. “If she’s at home then she’ll come to the door. She doesn’t come to the door. Therefore she isn’t at home.”

\[ H \rightarrow D, \sim D \mid \sim H \]

1. \[ H \rightarrow D \] P
2. \[ \sim D \] P
3. \[ \sim H \] 1, 2 MT

This argument is valid.
IV. “If she’s at home then she’ll come to the door. She doesn’t come to the door. If she’s taking the day off then she’s at home. Therefore she isn’t taking the day off.”

\[ H \rightarrow D, \sim D, O \rightarrow H \vdash \sim O \]

1. \[ H \rightarrow D \quad P \]
2. \[ \sim D \quad P \]
3. \[ O \rightarrow H \quad P \]
4. \[ \sim H \quad 1, 2 \text{ MT} \]
5. \[ \sim O \quad 3, 4 \text{ MT} \]

This argument is valid.

Some Skills from Formal Logic

Symbolizing Arguments and Constructing Proofs using

*Modus Ponens and Modus Tollens*
Valid Inference Forms

| P → Q, P |- Q | If Max is a poodle then Max is a dog. Max is a poodle. Therefore Max is a dog. | Modus Ponens (MP) |
| P → Q, Opp Q |- Opp P | If Max is a poodle then Max is a dog. Max is a not a dog. Therefore Max is not a poodle. | Modus Tollens (MT) |

V. “Reasonable people can disagree. If reasonable people can disagree then rationality isn’t completely objective. If rationality is like mathematics then it is completely objective. Therefore rationality isn’t like mathematics.”

D, D → ~O, M → O |- ~M

1. D P
2. D → ~O P
3. M → O P
4. ~O 1, 2 MP
5. ~M 3, 4 MT

This argument is valid.
Some Skills from Formal Logic

The Fallacy of Assuming the Consequent
And
The Fallacy of Negating the Antecedent

Invalid Inference Forms

| $P \rightarrow Q$, $Q |- P$ | If Max is a poodle then Max is a dog. Max is a dog. Therefore Max is a poodle. | Fallacy of Assuming the Consequent (FAC) |
|--------------------------|---------------------------------------------------------------------------------|----------------------------------|
| $P \rightarrow Q$, Opp $P |- Opp Q$ | If Max is a poodle then Max is a dog. Max is not a poodle. Therefore Max is not a dog. | Fallacy of Negating the Antecedent (FNA) |

Think clearly!
VI. “If the course isn’t easy then we’ll need to study. We don’t need to study. If the course requires very little reading then it’s easy. Therefore the course requires very little reading.”

\[ \neg E \rightarrow S, \neg S, L \rightarrow E \vdash L \]

1. \( \neg E \rightarrow S \) P
2. \( \neg S \) P
3. \( L \rightarrow E \) P
4. \( E \) 1, 2 MT
5. \( L \) 3, 4 FAC – Invalid Inference

This argument is invalid.

VII. “If he’s the best candidate then he’ll get the job. He’s the best candidate. If he doesn’t get the job then he won’t be happy. Therefore he’ll be happy.”

\[ B \rightarrow J, B, \neg J \rightarrow \neg H \vdash H \]

1. \( B \rightarrow J \) P
2. \( B \) P
3. \( \neg J \rightarrow \neg H \) P
4. \( J \) 1, 2 MP
5. \( H \) 3, 4 FNA – Invalid Inference

This argument is invalid.
VIII. “If God exists then there is no suffering in the world. There is suffering in the world. If God doesn’t exist then there is no life after death. Therefore there is no life after death.”

\[\begin{align*}
G \rightarrow \sim S, & \ S, \ \sim G \rightarrow \sim L \ | - \ \sim L \\
1. \ G \rightarrow \sim S & \quad P \\
2. \ S & \quad P \\
3. \ \sim G \rightarrow \sim L & \quad P \\
4. \ \sim G & \quad 1, 2 \ MT \\
5. \ \sim L & \quad 3, 4 \ MP \\
\end{align*}\]

This argument is valid.

This tells us \textit{nothing} about the truth of the premises.

This tells us \textit{nothing} about the truth of the conclusion.

This \textit{does} tell us that anyone who believes the premises is logically compelled to believe the conclusion.
IX. “Many atheists are rational people. If many atheists are rational people then there isn’t overwhelming evidence for God’s existence. If God doesn’t exist then there wouldn’t be overwhelming evidence for God’s existence. Therefore God doesn’t exist.”

\[ R, R \rightarrow \sim O, \sim G \rightarrow \sim O \vdash \sim G \]

1. \( R \) \hspace{1cm} P
2. \( R \rightarrow \sim O \) \hspace{1cm} P
3. \( \sim G \rightarrow \sim O \) \hspace{1cm} P
4. \( \sim O \) \hspace{1cm} 1, 2 MP
5. \( \sim G \) \hspace{1cm} 3, 4 FAC – Invalid Inference

This argument is invalid. The conclusion doesn’t follow from the premises.

X. “If some events violate natural laws then miracles occur. Some events do violate natural laws. If God didn’t exist then no miracles would occur. Therefore God exists.”

\[ V \rightarrow M, V, \sim G \rightarrow \sim M \vdash G \]

1. \( V \rightarrow M \) \hspace{1cm} P
2. \( V \) \hspace{1cm} P
3. \( \sim G \rightarrow \sim M \) \hspace{1cm} P
4. \( M \) \hspace{1cm} 1, 2 MP
5. \( G \) \hspace{1cm} 3, 4 MT

This argument is valid. This tells us nothing about the truth of the premises. This tells us nothing about the truth of the conclusion. This does tell us that anyone who believes the premises is logically compelled to believe the conclusion.
XI. “The conditions necessary for life are very precise. If life were the result of chance then the conditions necessary for life wouldn’t be very precise. If life were the result of chance then God wouldn’t exist. Therefore God does exist.”

\[ P, C \rightarrow \neg P, C \rightarrow \neg G \vdash G \]

1. \( P \) \( P \)
2. \( C \rightarrow \neg P \) \( P \)
3. \( C \rightarrow \neg G \) \( P \)
4. \( \neg C \) \( 1, 2 \) MT
5. \( G \) \( 3, 4 \) FNA – Invalid Inference

This argument is invalid.

The conclusion doesn’t follow from the premises.
Continue studying Formal Logic

1) Internet Resources
   Search under “Formal Logic” or “Symbolic Logic.”

2) Textbooks
   Used textbooks are inexpensive. Older editions are fine.

3) Courses
   Explore what’s available at your nearest college or university.
   Look especially at the philosophy (or mathematics) departments.
   Feel free to visit the professor and ask to see his or her syllabus.

When you study Formal Logic, you’ll learn

1) How to symbolize many different sorts of sentences.
2) How to recognize and use a wealth of other valid inference forms to construct proofs.

Why this is Useful

Being able to construct proofs – either on paper or in your head – makes it much easier to assess complex arguments for validity and to ensure that your own arguments are valid.

Formal logic is a power pill for the mind, and it’s also surprisingly fun! Enjoy it. 😊