Here, you’ll learn:

How to understand complex sentences by
- symbolizing singular statements
- symbolizing universal generalizations
- symbolizing existential generalizations

How to assess arguments for validity / invalidity by
- establishing that arguments are valid using UO, UI, EO, EI, QE
- establishing that arguments are invalid by constructing diagrams or interpretations (optional)

The Need for Predicate Logic

Consider the following argument

“All philosophers are fun at parties. Jennifer is a philosopher. Therefore, Jennifer is fun at parties.”

If the premises were true, would the conclusion have to be true too?
Yes, so this argument is valid.

In propositional logic, this argument would be symbolized like so:

“All philosophers are fun at parties. Jennifer is a philosopher. Therefore, Jennifer is fun at parties.”

P = Jennifer is a philosopher.
A = All philosophers are fun at parties.
F = Jennifer is fun at parties.

A, P ⊢ F

We can’t prove this argument in propositional logic. But, as we’ve seen, the English argument is valid. What we need, then, is another, more precise, way to symbolize this argument, a way that better exposes its structure and allows us to prove it.

We need predicate logic.

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1 Pospesel, *Predicate Logic*
Consider the following argument

“Jennifer is a philosopher. Jennifer is fun at parties. Therefore, some philosophers are fun at parties.”

If the premises were true, would the conclusion have to be true too?
Yes, so this argument is valid.

In propositional logic, this argument would be symbolized like so:

“Jennifer is a philosopher. Jennifer is fun at parties. Therefore, some philosophers are fun at parties.”

\[ P = \text{Jennifer is a philosopher.} \]
\[ F = \text{Jennifer is fun at parties.} \]
\[ S = \text{Some philosophers are fun at parties.} \]

\[ P, F \vdash S \]

We can’t prove this argument in propositional logic. But, as we’ve seen, the English argument is valid. What we need, then, is another, more precise, way to symbolize this argument, a way that better exposes its structure and allows us to prove it.

We need predicate logic.

**Statements in Predicate Logic**

There are three kinds of statements in predicate logic:

1) **Singular statements** that attribute properties to specific, named individuals (e.g. “Jennifer is a philosopher,” and “Jennifer is fun at parties.”)

They are symbolized using

1. **singular terms**, or names, that identify individuals (like “Jennifer”)
   These are small letters “a” through “w.” (“x,” “y,” and “z” have a special role that we’ll see soon.)
2. **predicate terms** that ascribe properties (like “is a philosopher.”)
   These are capital letters “A” through “Z.”
3. **connectors** from propositional logic (sometimes)

Examples:

- “Pa” is read as “a is P.”
- “¬Pa” is read as “a isn’t P.”
- “Pa & Qa” is read as “a is P and Q.”
“Pa ∨ Qa” is read as “Either a is P or a is Q.”
“Pa → Qa” is read as “If a is P then a is Q.”
“Pa ↔ Qa” is read as “a is P if and only if a is Q.”

2) Universal Generalizations that attribute properties to all individuals, or to all individuals in a certain category (e.g. “All philosophers are fun at parties.”)

They are symbolized using
1. **variables** that refers to unspecified individuals.
   These are small letters “x,” “y,” or “z.” (This is that special role mentioned above.)
2. **universal quantifiers**, “(x),” (or “(y)” or “(z)”), that are read as “for all x,” (or “for all y,” or “for all z,”) and that universal generalizations take as the main connector.
3. **predicate terms** that ascribe properties (like “is a philosopher.”)
   These are capital letters “A” through “Z.”
4. **connectors** from propositional logic (sometimes)

Examples:
“(x)Px” is read as “For all x, x is P.”
“(x)¬Px” is read as “For all x, x isn’t P.”
“(x)(Px & Qx)” is read as “For all x, x is P and Q.”
“(x)(Px ∨ Qx)” is read as “For all x, either x is P or x is Q.”
“(x)(Px → Qx)” is read as “For all x, if x is P then x is Q.”
“(x)(Px ↔ Qx)” is read as “For all x, x is P if and only if x is Q.”

3) Existential Generalizations that attribute properties to one or more unnamed individuals, or to one or more individuals in a certain category (e.g. “Some philosophers are fun at parties.”)

They are symbolized using
1. **variables** that refers to unspecified individuals.
   These are small letters “x,” “y,” or “z.”
2. **existential quantifiers**, “(∃x),” (or “(∃y)” or “(∃z)”), that are read as “there exists an x,” (or “there exists a y,” or “there exists a z,”) and that existential generalizations take as the main connector.
3. **predicate terms** that ascribe properties (like “is a philosopher.”)
   These are capital letters “A” through “Z.”
4. **connectors** from propositional logic (sometimes)

Examples:
“(∃x)Px” is read as “There exists an x that’s P.”
“(∃x)¬Px” is read as “There exists an x that isn’t P.”
“(∃x)(Px & Qx)” is read as “There exists an x that’s P and Q.”
“(∃x)(Px ∨ Qx)” is read as “There exists an x that’s either P or Q.”
“(∃x)(Px → Qx)” is read as “There exists an x such that if x is P then x is Q.”
“(∃x)(Px ↔ Qx)” is read as “There exists an x such that if x is P if and only if x is Q.”

Philosophical Observation: Note that “exists” is not a predicate but a quantifier. This has philosophical implications that appear most clearly – and influentially – in Kant’s criticism of the ontological argument.

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Working with Universal Generalizations: Symbolizing and the Inference Rule UO

Symbolizing: “All P are Q” will be symbolized as “(x)( Px → Qx)”

Think about this. Does this make sense to you?

To symbolize such sentences, ask yourself the following questions:

1) Are we making a claim about all members of a certain group?
   If so, you’ll use a “(x)” and almost always “(x)” is followed by a conditional, like so “(x)( → )”
2) About what group am I making my universal claim?
   This becomes the antecedent predicate.
   “(x)(Px → )”
3) What am I saying about all members of this group?
   This becomes the consequent predicate.
   “(x)(Px → Qx)”

We can’t use → O on lines like “(x)( Px → Qx)” because “→” isn’t the main connector.
“(x)” is the main connector. We need another rule to deal with lines that have “(x)” as the main connector.

UO (Universal Quantifier Out): From a line of the form (x)Px we may derive a line of the form Pn where “n” is any name.

Remember that we can apply UO only to universal generalizations (e.g. to (x)(Px → Qx) but not to (x)Px → (x)Qx.).

UO is valid because if a certain property is true of every individual then it’s true of any particular individual. We will, in fact, take this as a primitive inference rule.

UO is needed because it transforms a line from a formula in predicate logic to a formula in propositional logic, which we can manipulate using our propositional inference rules.
Symbolize and prove:

1. “All philosophers are fun at parties. Jennifer is a philosopher. Therefore, Jennifer is fun at parties.”

\[(x)(Px \rightarrow Fx), Pj \vdash Fj\]

1. \((x)(Px \rightarrow Fx)\) \hspace{1cm} P
2. Pj \hspace{1cm} P – want \ Fj
3. Pj \rightarrow Fj \hspace{1cm} 1 \text{ UO}
4. Fj \hspace{1cm} 2,3 \rightarrow O

2. “Every philosopher is fun at parties. Jennifer isn’t fun at parties. Thus, Jennifer isn’t a philosopher.”

\[(x)(Px \rightarrow Fx), \neg Fj \vdash \neg Pj\]

1. \((x)(Px \rightarrow Fx)\) \hspace{1cm} P
2. \neg Fj \hspace{1cm} P - want \neg Pj
3. Pj \rightarrow Fj \hspace{1cm} 1 \text{ UO}
4. \neg Pj \hspace{1cm} 2, 3 \text{ MT}

3. “Any philosopher is fun to have at a party. People who are fun to have at parties are charming. Bill is a philosopher. Thus, Bill is charming.”

\[(x)(Px \rightarrow Fx), (x)(Fx \rightarrow Cx), Pb \vdash Cb\]

1. \((x)(Px \rightarrow Fx)\) \hspace{1cm} P
2. \((x)(Fx \rightarrow Cx)\) \hspace{1cm} P
3. Pb \hspace{1cm} P - want Cb
4. Pb \rightarrow Fb \hspace{1cm} 1 \text{ UO}
5. Fb \rightarrow Cb \hspace{1cm} 2 \text{ UO}
6. Pb \rightarrow Cb \hspace{1cm} 4, 5 \text{ CH}
7. Cb \hspace{1cm} 3, 6 \rightarrow O
4. “Philosophers are fun to have at parties and good with small animals. Bob isn’t good with small animals. Thus, Bob isn’t a philosopher.”

\[(x)(Px \rightarrow (Fx \land Gx)), \sim Gb \vdash \sim Pb\]

1. \( (x)(Px \rightarrow (Fx \land Gx)) \) \( P \)  
2. \( \sim Gb \) \( P - \text{want } \sim Pb \)  
3. \( Pb \rightarrow (Fb \land Gb) \) \( 1 \text{ UO} \)  
4. \( \sim(Fb \land Gb) \rightarrow \sim Pb \) \( 3 \text{ CN} \)  
5. \( (\sim Fb \lor \sim Gb) \rightarrow \sim Pb \) \( 4 \text{ DM} \)  
6. \( \sim Fb \lor \sim Gb \) \( 2 \text{ v I} \)  
7. \( \sim Pb \) \( 5,6 \rightarrow O \)

Symbolizing: “Only P are Q” will be symbolized as “\((x)(Qx \rightarrow Px)\)”

Think about this. Does this make sense to you?
If it doesn’t entirely make sense, think of it this way:
“Only P are Q.”
“Only if something is a P can it be a Q.”
“Something is a Q only if it’s a P.”
“(x)(Qx \rightarrow Px)”

Symbolize and prove:
5. “Only philosophers read Frege. Lisa reads Frege. Thus, Lisa is a philosopher.”

\[(x)(Fx \rightarrow Px), Fl, \vdash Pl\]

1. \( (x)(Fx \rightarrow Px) \) \( P \)  
2. \( Fl \) \( P - \text{want } Pl \)  
3. \( Fl \rightarrow Pl \) \( 1 \text{ UO} \)  
4. \( Pl \) \( 2, 3 \rightarrow O \)

6. “Only books that provide us with scientific truths are important. The Bible doesn’t provide us with scientific truths. Therefore, the Bible isn’t important.”

\[(x)(Ix \rightarrow Sx), \sim Sb \vdash \sim Ib\]

1. \( (x)(Ix \rightarrow Sx) \) \( P \)  
2. \( \sim Sb \) \( P - \text{want } \sim Ib \)  
3. \( Ib \rightarrow Sb \) \( 1 \text{ UO} \)  
4. \( \sim Ib \) \( 2,3 \text{ MT} \)
7. “All ethicists are either consequentialists or deontologist. Only ethicists are able to think clearly about the death penalty. All consequentialists are ethical naturalists. Only people who believe in rights are deontologists. Therefore, if George can think clearly about the death penalty, he’s an ethical naturalist or believes in rights.”

\[(x)(Ex \rightarrow (Cx \lor Dx)), (x)(Tx \rightarrow Ex), (x)(Cx \rightarrow Nx), (x)(Dx \rightarrow Rx) \vdash Tg \rightarrow (Ng \lor Rg)\]

1. \( (x)(Ex \rightarrow (Cx \lor Dx)) \)  P
2. \( (x)(Tx \rightarrow Ex) \)  P
3. \( (x)(Cx \rightarrow Nx) \)  P
4. \( (x)(Dx \rightarrow Rx) \)  P - want \( Tg \rightarrow (Ng \lor Rg) \)
5. \( Tg \)  PP – want \( Ng \lor Rg \)

1. \( Eg \rightarrow (Cg \lor Dg) \)  1 UO
2. \( Tg \rightarrow Eg \)  2 UO
3. \( Cg \rightarrow Ng \)  3 UO
4. \( Dg \rightarrow Rg \)  4 UO
5. \( Tg \rightarrow (Cg \lor Dg) \)  6, 7 CH
1,2,5  11. \( Cg \lor Dg \)  5, 10 \( \rightarrow \)O
1,2,3,4,5  12. \( Ng \lor Rg \)  8,9,11 CD
1,2,3,4  13. \( Tg \rightarrow (Ng \lor Rg) \)  5 – 12 \( \rightarrow \)I

Symbolizing: “No P are Q” is symbolized as “\((x)(Px \rightarrow \sim Qx)\)”

Think about this. Does this make sense to you?

Symbolize and prove:

8. “No theologians are atheists. All Philosophers are atheists. Thus, either Sam isn’t a theologian or Sam isn’t a philosopher.”

\[(x)(Tx \rightarrow \sim Ax), (x)(Px \rightarrow Ax) \vdash \sim Ts \lor \sim Ps\]

1. \( (x)(Tx \rightarrow \sim Ax) \)  P
2. \( (x)(Px \rightarrow Ax) \)  P - want \( \sim Ts \lor \sim Ps \)
3. \( Ts \rightarrow \sim As \)  1 UO
4. \( Ps \rightarrow As \)  2 UO
5. \( \sim As \rightarrow \sim Ps \)  4 CN
6. \( Ts \rightarrow \sim Ps \)  3, 5 CH
7. \( \sim Ts \lor \sim Ps \)  6 AR
9. “Only philosophers read Frege. No philosophers have a sense of humor. Only people with a sense of humor get invited to the best parties. Janet reads Frege or Buber. Janet gets invited to the best parties. Thus, Janet reads Buber.”

\[(x)(Fx \rightarrow Px), (x)(Px \rightarrow \sim Hx), (x)(Ix \rightarrow Hx), Fj \lor Bj, Ij \vdash Bj\]

1. \[(x)(Fx \rightarrow Px)\]  P
2. \[(x)(Px \rightarrow \sim Hx)\]  P
3. \[(x)(Ix \rightarrow Hx)\]  P
4. \[Fj \lor Bj\]  P
5. \[Ij\]  P – want Bj
6. \[Fj \rightarrow Pj\]  1 UO
7. \[Pj \rightarrow \sim Hj\]  2 UO
8. \[Ij \rightarrow Hj\]  3 UO
9. \[Hj\]  5, 8 \(\rightarrow\) O
10. \[\sim Pj\]  7, 9 MT
11. \[\sim Fj\]  6, 10 MT
12. \[Bj\]  4, 11 vo

10. (Optional) “No philosophers read the horoscopes. No theologians read the funnies. Martha reads both the horoscopes and the funnies. Thus, Martha isn’t a philosopher and she isn’t a theologian.”

\[(x)(Px \rightarrow \sim Hx), (x)(Tx \rightarrow \sim Fx), Hm \& Fm \vdash \sim Pm \& \sim Tm\]

1. \[(x)(Px \rightarrow \sim Hx)\]  P
2. \[(x)(Tx \rightarrow \sim Fx)\]  P
3. \[Hm \& Fm\]  P - want \(\sim Pm \& \sim Tm\)
4. \[Pm \rightarrow \sim Hm\]  1 UO
5. \[Tm \rightarrow \sim Fm\]  2 UO
6. \[Hm\]  3 \&O
7. \[Fm\]  3 \&O
8. \[\sim Pm\]  4, 6 MT
9. \[\sim Tm\]  5, 7 MT
10. \[\sim Pm \& \sim Tm\]  6, 8 \&I
11. (Optional) “No philosophers believe in magic. Only people who believe in magic can believe in God. All theologians believe in God. Thus, Ellen is a philosopher only if she isn’t a theologian.

\[(x)(P \rightarrow \sim Mx), (x)(Gx \rightarrow Mx), (x)(Tx \rightarrow Gx) \vdash Pe \rightarrow \sim Te\]

1. \((x)(P \rightarrow \sim Mx)\) \hspace{1cm} P
2. \((x)(Gx \rightarrow Mx)\) \hspace{1cm} P
3. \((x)(Tx \rightarrow Gx)\) \hspace{1cm} P - want \(Pe \rightarrow \sim Te\)
4. \(Pe \rightarrow \sim Me\) \hspace{1cm} 1 UO
5. \(Ge \rightarrow Me\) \hspace{1cm} 2 UO
6. \(Te \rightarrow Ge\) \hspace{1cm} 3 UO
7. \(\sim Me \rightarrow \sim Ge\) \hspace{1cm} 5 CN
8. \(Pe \rightarrow \sim Ge\) \hspace{1cm} 4, 7 CH
9. \(\sim Ge \rightarrow \sim Te\) \hspace{1cm} 6 CN
10. \(Pe \rightarrow \sim Te\) \hspace{1cm} 8, 9 CH

Symbolizing: “Not all P are Q” is symbolized as “\(\sim(x)(P \rightarrow Qx)\)”

Think about this. Does this make sense to you?

Symbolize and prove:

12. “Sam is a philosopher and a theologian. Sam isn’t fun at parties. Thus, not all philosophers are fun at parties.”

\[Ps \& Ts, \sim Fs \vdash \sim(x)(P \rightarrow Fx)\]

1 \hspace{1cm} 1. Ps \& Ts \hspace{1cm} P
2 \hspace{1cm} 2. \sim Fs \hspace{1cm} P - want \(\sim(x)(P \rightarrow Fx)\)
3 \hspace{1cm} 3. \((x)(P \rightarrow Fx)\) \hspace{1cm} PP – want contradiction
3 \hspace{1cm} 4. Ps \rightarrow Fs \hspace{1cm} 3 UO
1 \hspace{1cm} 5. Ps \hspace{1cm} 1 &O
1,3 \hspace{1cm} 6. Fs \hspace{1cm} 4, 5 \rightarrow O
1,2,3 \hspace{1cm} 7. Fs \& \sim Fs \hspace{1cm} 2, 6 &I
1,2 \hspace{1cm} 8. \sim(x)(P \rightarrow Fx) \hspace{1cm} 3 – 7 \sim I
13. “Sam is either a philosopher or a theologian. All theologians are sanctimonious. Sam is neither sanctimonious nor critical. Thus, not all philosophers are critical.”

\[
\text{PsVTs, } (x)(T_x \rightarrow S_x), \sim S_s \& \sim C_s \vdash \sim (x)(P_x \rightarrow C_x)
\]

1. PsVTs \hspace{1cm} P
2. \((x)(T_x \rightarrow S_x)\) \hspace{1cm} P
3. \sim S_s \& \sim C_s \hspace{1cm} P \text{ - want } \sim (x)(P_x \rightarrow C_x)
4. \((x)(P_x \rightarrow C_x)\) \hspace{1cm} PP \text{ - want contradiction}
5. \(P_s \rightarrow C_s\) \hspace{1cm} 4 \text{ UO}
6. \(T_s \rightarrow S_s\) \hspace{1cm} 2 \text{ UO}
1,2,4 \hspace{1cm} 7. \(C_s \vee S_s\) \hspace{1cm} 1, 5, 6 \text{ CD}
1,2,4 \hspace{1cm} 8. \(S_s \vee C_s\) \hspace{1cm} 7 \text{ v Com}
3 \hspace{1cm} 9. \(\sim (S_s \vee C_s)\) \hspace{1cm} 3 \text{ DM}
1,2,3,4 \hspace{1cm} 10. \((S_s \& C_s) \& \sim (S_s \vee C_s)\) \hspace{1cm} 8, 9 \text{ &I}
1,2,3 \hspace{1cm} 11. \(\sim (x)(P_x \rightarrow C_x)\) \hspace{1cm} 4 – 10 \sim I

14. “All consequentialists take animal pleasure and pain into account. Only people who don’t take animal pleasure and pain into account could eat meat. Frank is a meat-eating ethicist. Thus, not all ethicists are consequentialists.”

\[
(x)(C_x \rightarrow A_x), (x)(M_x \rightarrow \sim A_x), \ Ef \& M_f \vdash \sim (x)(E_x \rightarrow C_x)
\]

1. \(1. (x)(C_x \rightarrow A_x)\) \hspace{1cm} P
2. \(2. (x)(M_x \rightarrow \sim A_x)\) \hspace{1cm} P
3. \(3. Ef \& M_f\) \hspace{1cm} P \text{ - want } \sim (x)(E_x \rightarrow C_x)
4. \(4. (x)(E_x \rightarrow C_x)\) \hspace{1cm} PP \text{ - want contradiction}
5. \(C_f \rightarrow A_f\) \hspace{1cm} 1 \text{ UO}
6. \(M_f \rightarrow \sim A_f\) \hspace{1cm} 2 \text{ UO}
7. \(E_f \rightarrow C_f\) \hspace{1cm} 4 \text{ UO}
1,4 \hspace{1cm} 8. \(E_f \rightarrow A_f\) \hspace{1cm} 5, 7 \text{ CH}
3 \hspace{1cm} 9. \(E_f\) \hspace{1cm} 3 \text{ &O}
1,3,4 \hspace{1cm} 10. \(A_f\) \hspace{1cm} 8, 9 \rightarrow O
3 \hspace{1cm} 11. \(M_f\) \hspace{1cm} 3 \text{ &O}
2,3 \hspace{1cm} 12. \(\sim A_f\) \hspace{1cm} 6, 11 \rightarrow O
1,2,3,4 \hspace{1cm} 13. \(A_f \& \sim A_f\) \hspace{1cm} 10, 12 \text{ &I}
1,2,3 \hspace{1cm} 14. \(\sim (x)(E_x \rightarrow C_x)\) \hspace{1cm} 4 – 13 \sim I
Optional: Symbolizing Non-Standard Universal Generalizations

1. “Everything is physical.”
   \((x)Px\)

2. “Nothing is physical.”
   \((x)\neg Px\)

3. “Everything is both physical and mental.”
   \((x)(Px \& Mx)\)

4. “Everything is either a physical thing or a mental event.”
   \((x)(Px \lor Mx)\)

5. “Either everything is a physical thing or everything is a mental event.”
   \((x)Px \lor (x)Mx\)

6. “Things are mental events if they aren’t physical.”
   \((x)(\neg Px \rightarrow Mx)\)

7. “All physical things are extended and divisible.”
   \((x)(Px \rightarrow (Ex \& Dx))\)

8. “All extended and divisible things are physical.”
   \((x)((Ex \& Dx) \rightarrow Px)\)

9. “All mental events are either phenomenological or intentional.”
   \((x)(Mx \rightarrow (Px \lor Ix))\)

10. “Every physical thing is extended, but no mental event is.”
    \((x)(Px \rightarrow Ex) \& (x)(Mx \rightarrow \neg Ex)\)

11. “If everything is nonphysical then everything is a mental event.”
    \((x)\neg Px \rightarrow (x)Mx\)

12. “If all physical things are extended, then no physical thing is a mental event.”
    \((x)(Px \rightarrow Ex) \rightarrow (x)(Px \rightarrow \neg Mx)\)

13. “All mental events have a truth value, provided that they have content.”
    \((x)(Mx \rightarrow Cx) \rightarrow (x)(Mx \rightarrow Tx)\)

14. “Philosophers of mind and philosophers of language are interested in formal logic.”
    \((x)((Mx \& Lx) \rightarrow Ix)\)
15. “Philosophers who believe that mental events are nonphysical are generally taunted unless they’re very good.”

\((x)(Bx \rightarrow (\neg Gx \rightarrow Tx))\)

Quiz time.

Working with Existential Generalizations: Symbolizing and the Inference Rule EI

Symbolizing: “Some P are Q” will be symbolized as “\((\exists x)(Px & Qx)\)”

Think about this. Does this make sense to you?

To symbolize such sentences, ask yourself the following questions:

1) Are we making a claim about some but not all members of a certain group?
   If so, you’ll use a “\((\exists x)\)” and almost always “\((\exists x)\)” is followed by a conjunction, like so “\((\exists x)( & )\)”
2) About what group am I making my existential claim?
   This becomes the first conjunct predicate.
   “\((\exists x)(Px & )\)”
3) What am I saying about some members of this group?
   This becomes the second conjunct predicate.
   “\((\exists x)(Px & Qx)\)”

We can’t use &I to get lines like “\((\exists x)(Px & Qx)\)” because “&” isn’t the main connector.
“\((\exists x)\)” is the main connector so we need another rule to get lines that have “\((\exists x)\)” as the main
c

EI (Existential Quantifier In): From a line of the form Pn, where “n” is any name, we may derive a line of the form \((\exists x)Px\).

EI is valid because if a given individual has a certain property then clearly there exists at least one individual with that property. We’ll take this as a primitive inference rule, too.

Symbolize and prove:

1. “Jennifer is a philosopher. Jennifer is fun at parties. Therefore, some philosophers are fun at parties.”

\[
Pj, Fj \vdash (\exists x)(Px & Fx)
\]

1. \(Pj\)  
   P
2. \(Fj\)  
   P - want \((\exists x)(Px & Fx)\)
3. Pj & Fj    1, 2 &I  
4. (∃x)(Px & Fx)  3 EI

2. “Maria is a logician and a theist. All logicians are rigorous thinkers. No theists are cruel. Therefore some rigorous thinkers aren’t cruel.”

Lm & Tm, (x)(Lx → Rx), (x)(Tx → ~Cx) ⊢ (∃x)(Rx & ~Cx)

1. Lm & Tm        P
2. (x)(Lx → Rx)   P
3. (x)(Tx → ~Cx)  P - want (∃x)(Rx & ~Cx)
4. Lm → Rm        2 UO
5. Tm → ~Cm       3 UO
6. Lm            1 &O
7. Tm            2 &O
8. Rm            4, 6 →O
9. ~Cm           5, 7 →O
10. Rm & ~Cm     8, 9 &I
11. (∃x)(Rx & ~Cx) 10 EI

3. “Everything is physical. Everything has a soul. Therefore some physical things have souls.”

(x)Px, (x)Sx ⊢ (∃x)(Px & Sx)

1. (x)Px        P
2. (x)Sx        P - want (∃x)(Px & Sx)
3. Pa           1 UO
4. Sa           2 UO
5. Pa & Sa      3, 4 &I
6. (∃x)(Px & Sx) 5 EI
4. “Everyone is either a scientist or an artist at heart. Only emotional people are artists. Only rational people are scientists. Therefore, someone is either emotional or rational.”

\[(x)(Sx \lor Ax), (x)(Ax \rightarrow Ex), (x)(Sx \rightarrow Rx) \vdash (\exists x)(Ex \lor Rx)\]

1. \((x)(Sx \lor Ax)\)  P
2. \((x)(Ax \rightarrow Ex)\)  P
3. \((x)(Sx \rightarrow Rx)\)  P - want \((\exists x)(Ex \lor Rx)\)
4. \(Sa \lor Aa\)  1 UO
5. \(Aa \rightarrow Ea\)  2 UO
6. \(Sa \rightarrow Ra\)  3 UO
7. \(Ea \lor Ra\)  4, 5, 6 CD
8. \((\exists x)(Ex \lor Rx)\)  7 EI

5. “Philip is both an athlete and scholar. If someone is both an athlete and a scholar then anybody who’s self-disciplined can excel in a variety of fields. Anyone who can excel in a variety of fields can be popular. Therefore if Jennifer is self-disciplined, she can be popular.”

\[Ap \& Sp, (\exists x)(Ax \& Sx) \rightarrow (x)(Dx \rightarrow Vx), (x)(Vx \rightarrow Px) \vdash Dj \rightarrow Pj\]

1. \(Ap \& Sp\)  P
2. \((\exists x)(Ax \& Sx) \rightarrow (x)(Dx \rightarrow Vx)\)  P
3. \((x)(Vx \rightarrow Px)\)  P – want \(Dj \rightarrow Pj\)
4. \((\exists x)(Ax \& Sx)\)  1 EI
5. \((x)(Dx \rightarrow Vx)\)  2, 4 \(\rightarrow\)O
6. \(Vj \rightarrow Pj\)  3 UO
7. \(Dj \rightarrow Vj\)  5 UO
8. \(Dj \rightarrow Pj\)  6, 7 CH

Optional: More Symbolizing

1. “Almost all physical things are extended.”
\[(\exists x)(Px \& Ex)\]

2. “Something is a mental event.”
\[(\exists x)Mx\]

3. “Something isn’t a mental event.”
\[(\exists x)\sim Mx\]

4. “It isn’t the case that something is a mental event.”
\[(\exists x)\sim Mx\]
5. “Some extended divisible things aren’t physical.”
\((\exists x)((Ex & Dx) & \neg Px)\)

6. “Something is neither physical nor a mental event.”
\((\exists x)(\neg Px & \neg Mx)\)

7. “There are mental events only if there are nonphysical events.”
\((\exists x)Mx \rightarrow (\exists x)\neg Px\)

8. “If any philosopher is a physicalist then some philosopher is a spiritualist.”
\((\exists x)(Hx & Px) \rightarrow (\exists x)(Hx & Sx)\)

9. “If any philosopher believes that mental events are nonphysical, it’s Jim.”
\((\exists x)(Hx & Bx) \rightarrow (Hj & Bj)\)

10. “Even though everything is physical, something is a mental event.”
\((x)Px & (\exists x)Mx\)

11. “Something is mental unless everything is physical.”
\(\neg(x)Px \rightarrow (\exists x)Mx\)

12. “If some mental events are physical, then they all are.”
\((\exists x)(Mx & Px) \rightarrow (x)(Mx \rightarrow Px)\)

Quiz time.

Three More Inference Rules – EO, UI, QE

So far, we have UO and EI.
We’re missing EO and UI.

Can we just help ourselves to these rules? What would they be?

Figuring out EO

EO would tell us to go from a line of the form \((\exists x)(Px)\) to a line of the form \(Pn\), where n is a name. Is that okay? Let’s see.
Consider the following argument:
“Somebody is a serial killer. Therefore Dona Warren is a serial killer.”

\( (\exists x)Kx \vdash Kd \)

1. \( (\exists x)Kx \) \hspace{1cm} P \hspace{1cm} \text{- True}
2. \( Kd \) \hspace{1.5cm} 1 \text{ EO} \hspace{1cm} \text{- False!}

Since we’ve gone from true to false, some inference is invalid and there’s only one inference here - our EO - so that’s got to be where the problem is.

Now consider this argument:
“Some parents are female. Some parents are male. Therefore some females are male.”

\( (\exists x)(P_x \& F_x), (\exists x)(P_x \& M_x) \vdash (\exists x)(F_x \& M_x) \)

1. \( (\exists x)(P_x \& F_x) \) \hspace{1.5cm} P \hspace{1.5cm} \text{- True}
2. \( (\exists x)(P_x \& M_x) \) \hspace{1.5cm} P \hspace{1.5cm} \text{- True}
3. \( Pa \& Fa \) \hspace{1.5cm} 1 \text{ EO} 
4. \( Pa \& Ma \) \hspace{1.5cm} 2 \text{ EO} 
5. \( Fa \) \hspace{1.5cm} 3 \text{ &O} 
6. \( Ma \) \hspace{1.5cm} 4 \text{ &O} 
7. \( Fa \& Ma \) \hspace{1.5cm} 5,6 \text{ &I} 
8. \( (\exists x)(F_x \& M_x) \) \hspace{1.5cm} 7 \text{ EI} \hspace{1cm} \text{-False!}

Since we’ve gone from true to false, some inference is invalid. &O, &I and EI are already known to be fine, so the problem must once again be with EO.

But what is the problem with EO, as we’ve stated it? Intuitively, the problem is that the rule, as we’ve stated it, allows us to go from making a claim about someone-we-don’t-know-who to making that claim about somebody in particular.

In argument 1, we went from claiming that someone is a serial killer to claiming that I am a serial killer.

\( (\exists x)Kx \vdash Kd \)

1. \( (\exists x)Kx \) \hspace{1cm} P 
2. \( Kd \) \hspace{1.5cm} 1 \text{ EO} \hspace{1cm} \leftarrow \text{ This move shouldn’t be allowed.}
In argument 2, we went from the claim that *someone* is both a parent and a female and the claim that *someone* is both a parent and a male to the claim that whomever is both a parent and a female is also the person who’s a parent and a male. *That’s* what went wrong.

\[(\exists x)(P_x \& F_x), (\exists x)(P_x \& M_x) \vdash (\exists x)(F_x \& M_x)\]

1. \[(\exists x)(P_x \& F_x)\] P
2. \[(\exists x)(P_x \& M_x)\] P
3. \[P_a \& F_a\] 1 EO
4. \[P_a \& M_a\] 2 EO \(\leftarrow\) This move shouldn’t be allowed.
5. \[F_a\] 3 &O
6. \[M_a\] 4 &O
7. \[F_a \& M_a\] 5,6 &I
8. \[(\exists x)(F_x \& M_x)\] 7 EI

It’s all well and good to invent a name to stand in as a place holder for someone we know must exist but are unable to identify. That’s what the name “Jack the Ripper” does. Instead of talking about “The person who went about killing prostitutes in London’s East End in the late 1800’s,” we can say “Oh, let’s just call him ‘Jack the Ripper.’” That’s okay. It’s *not* okay to say “Oh, let’s just call him ‘the Prince of Wales,’” or some other specific person who was around at the time. (Compare to what we did when proving argument 1.)

And if, later on, we want to talk about someone who was burgling the homes of the aristocracy in Victorian London, we can’t use “Jack the Ripper” to refer to that person, because we’ve already used “Jack the Ripper” to refer to the notorious serial killer and we don’t want to imply that the burglar and the murderer are the same person. (Compare to what we did when proving argument 2.)

All this means that we need to put some restrictions on EO.

**EO (Existential Quantifier Out):** From a line of the form \((\exists x)P_x\), we may derive a line of the form \(P_n\), where “\(n\)” is any name *that is both new to the argument and new to the proof.*

Now, see how our application of EO in argument 1 was wrong? We can’t use “d” in EO in line 2, because it isn’t new to the argument; “d” already refers to me and it appears in the symbolization of the ultimate conclusion.

\[(\exists x)K_x \vdash K_d\]

1. \[(\exists x)K_x\] P
2. \[K_d\] 1 EO \(\leftarrow\) This move isn’t allowed.

And see how the use of EO in argument 2 is mistaken? Once “a” has been used on line 3, it isn’t new to the proof and so it can’t be used for EO again on line 4.


\((\exists x)(P x \& F x), (\exists x)(P x \& M x) \vdash (\exists x)(F x \& M x)\)

1. \((\exists x)(P x \& F x)\)  \(P\)
2. \((\exists x)(P x \& M x)\)  \(P\)
3. \(P a \& Fa\)  1 EO
4. \(P a \& Ma\)  2 EO  \(\text{This move isn’t allowed.}\)
5. \(F a\)  3 &O
6. \(M a\)  4 &O
7. \(F a \& Ma\)  5,6 &I
8. \((\exists x)(F x \& M x)\)  7 EI

Now we have an adequate EO rule.

EO is similar in function to UO in that both rules transform a line in predicate logic to a line in propositional logic, which we can then manipulate with our propositional inference rules.

Also, like UO, we can apply EO only to the main connector, to a sentence like \((\exists x)(P x \& Q x)\) but not to a sentence. \((\exists x)Px \& (\exists x)Qx\).

EO differs from UO in that UO has no restrictions on the name that is used. (This makes sense when you think about it. If something is true of everyone then it’s true of anyone, including people we know or have already discussed.) EO, however, can use only new names. This is important to remember.

Because there are no restrictions on the name you choose for UO, if you need to do both EO and UO in a proof, it’s always wise to do EO first. Once you do EO, you can then do UO with the same name that you used for EO. If, on the other hand, you do UO first, the name won’t be new and you won’t be able to do EO with that name. You’ll need to choose another name for your application of EO, and that will probably screw up the proof.

Symbolize and prove:

1. “Atheists who believe in reincarnation do exist. Anyone who believes in reincarnation is a substance dualist. Therefore, some atheists are substance dualists.”

\((\exists x)(A x \& B x), (x)(B x \rightarrow D x) \vdash (\exists x)(A x \& D x)\)

1. \((\exists x)(A x \& B x)\)  \(P\)
2. \((x)(B x \rightarrow D x)\)  \(P\)  – want \((\exists x)(A x \& D x)\)
3. \(A a \& Ba\)  1 EO
4. \(B a \rightarrow Da\)  2 UO
5. \(A a\)  3 &O
6. \(B a\)  3 &O
7. \(Da\)  4, 6 \(\rightarrow\)O
8. Aa & Da 5, 7 &I
9. (∃x)(Ax & Dx) 8 EI

2. (Optional) “All philosophers think about big questions. No shallow people think about big questions. Some philosophy teachers are shallow people. Therefore, some philosophy teachers aren’t philosophers.”

(x)(Px → Qx), (x)(Sx → ~Qx), (∃x)(Tx & Sx) |- (∃x)(Tx & ~Px)
1. (x)(Px → Qx) P
2. (x)(Sx → ~Qx) P
3. (∃x)(Tx & Sx) P – want (∃x)(Tx & ~Px)
4. Ta & Sa 3 EO
5. Pa → Qa 1 UO
6. Sa → ~Qa 2 UO
7. Ta 4 &O
8. Sa 4 &O
9. ~Qa 6, 8 →O
10. ~Pa 5, 9 MT
11. Ta & ~Pa 7, 10 &I
12. (∃x)(Tx & ~Px) 11 EI

3. (Optional) “Some historians are talkative. All historians are intelligent. Therefore, some talkative people are intelligent.”

(∃x)(Hx & Tx), (x)(Hx → Ix) |- (∃x)(Tx & Ix)
1. (∃x)(Hx & Tx) P
2. (x)(Hx → Ix) P - want (∃x)(Tx & Ix)
3. Ha & Ta 1 EO (Note, this is done before UO!)
4. Ha → Ia 2 UO
5. Ha 3 &O
6. Ia 4,5 →O
7. Ta 3 &O
8. Ta & Ia 6,7 &I
9. (∃x)(Tx & Ix) 8 EI

4. “Someone is either making this story up or gullible enough to believe it. Anyone who would make this story up is dishonest. Anyone gullible enough to believe this story is disabled. Therefore, someone is either dishonest or disabled.”

(∃x)(Mx ∨ Gx), (x)(Mx → Hx), (x)(Gx → Ax) |- (∃x)(Hx ∨ Ax)
1. \((\exists x)(Mx \lor Gx)\) \hspace{1cm} P  
2. \((x)(Mx \rightarrow Hx)\) \hspace{1cm} P  
3. \((x)(Gx \rightarrow Ax)\) \hspace{1cm} P - want \((\exists x)(Hx \lor Ax)\)  
4. Ma v Ga \hspace{1cm} 1 EO  
5. Ma → Ha \hspace{1cm} 2 UO  
6. Ga → Aa \hspace{1cm} 3 UO  
7. Ha v Aa \hspace{1cm} 4,5,6 CD  
8. \((\exists x)(Hx \lor Ax)\) \hspace{1cm} 7 EI 

5. (Optional) “Some foreign films are entertaining” is logically equivalent to “Some entertaining films are foreign.”

\[(\exists x)(Fx & Ex) \equiv (\exists x)(Ex & Fx)\]

1. \((\exists x)(Fx & Ex)\) \hspace{1cm} P  
2. \((\exists x)(Ex & Fx)\) \hspace{1cm} 1 & COM  

1. \((\exists x)(Ex & Fx)\) \hspace{1cm} P  
2. \((\exists x)(Fx & Ex)\) \hspace{1cm} 1 & COM 

6. (Optional) “Someone is either lying or guilty” is logically equivalent to “Someone isn’t both honest and innocent.” (For our purposes, “honest” is “not lying” and “innocent” is “not guilty.”)

\[(\exists x)(Lx \lor Gx) \equiv (\exists x)\sim(\sim Lx \land \sim Gx)\]

1. \((\exists x)(Lx \lor Gx)\) \hspace{1cm} P  
2. \((\exists x)\sim(\sim Lx \land \sim Gx)\) \hspace{1cm} 1 DM  

1. \((\exists x)\sim(\sim Lx \land \sim Gx)\) \hspace{1cm} P  
2. \((\exists x)(Lx \lor Gx)\) \hspace{1cm} 1 DM
7. “Some philosophers are theists. No theists are convinced by the problem of evil. Therefore, not all philosophers are convinced by the problem of evil.”

$$(\exists x)(P_x \land T_x), (x)(T_x \rightarrow \neg C_x) \vdash \neg (x)(P_x \rightarrow C_x)$$

1  1. $$\exists x (P_x \land T_x)$$  P
2  2. $$(x)(T_x \rightarrow \neg C_x)$$  P - want $$\neg (x)(P_x \rightarrow C_x)$$
3  3. $$(x)(P_x \rightarrow C_x)$$  PP – want contradiction
1  4. $$P_a \land T_a$$ 1 EO
2  5. $$T_a \rightarrow \neg C_a$$ 2 UO
3  6. $$P_a \rightarrow C_a$$ 3 UO
1  7. $$P_a$$ 4 &O
1,3  8. $$C_a$$ 6, 7 $$\rightarrow$$O
1  9. $$T_a$$ 4 &O
1,2  10. $$\neg C_a$$ 5, 9 $$\rightarrow$$O
1,2,3  11. $$C_a$$ & $$\neg C_a$$ 8, 10 &I
1,2  12. $$\neg (x)(P_x \rightarrow C_x)$$ 3 – 11 $$\neg$$I

8. “All rationalists are adept at logic. But nobody adept at logic would be misled by the fallacy of assuming the consequent. Therefore, it’s false that there are rationalists who are misled by the fallacy of assuming the consequent.”

$$(x)(R_x \rightarrow A_x), (x)(A_x \rightarrow \neg M_x) \vdash \neg \exists x(R_x \land M_x)$$

1  1. $$(x)(R_x \rightarrow A_x)$$  P
2  2. $$(x)(A_x \rightarrow \neg M_x)$$  P - want $$\neg \exists x(R_x \land M_x)$$
3  3. $$\exists x(R_x \land M_x)$$  PP – want contradiction
3  4. $$R_a \land M_a$$ 3 EO
1  5. $$R_a \rightarrow Aa$$ 1 UO
2  6. $$Aa \rightarrow \neg Ma$$ 2 UO
3  7. $$R_a$$ 4 &O
3  8. $$Ma$$ 4 &O
1,3  9. $$Aa$$ 5,7 $$\rightarrow$$O
1,2,3  10. $$\neg Ma$$ 6, 9 $$\rightarrow$$O
1,2,3  11. $$Ma$$ & $$\neg Ma$$ 8, 10 &I
1,2  12. $$\neg \exists x(R_x \land M_x)$$ 3 – 11 $$\neg$$I
9. (Optional) “Some philosophers aren’t logicians. No philosophers are superstitious. Therefore it’s false that only logicians aren’t superstitious.”

\[(\exists x)(P x \land \neg L x), \quad (x)(P x \to \neg S x) \vdash \neg (x)(\neg S x \to L x)\]

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<td>1</td>
<td>1. (\exists x)(P x \land \neg L x)</td>
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<td>2</td>
<td>2. (x)(P x \to \neg S x)</td>
<td>P - want \neg (x)(\neg S x \to L x)</td>
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<td>3. (x)(\neg S x \to L x)</td>
<td>PP - want contradiction</td>
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<td>4. P a \land \neg L a</td>
<td>1 EO</td>
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<td>5. P a \to \neg S a</td>
<td>2 UO</td>
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<td>6. \neg S a \to L a</td>
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<td>2,3</td>
<td>7. P a \to L a</td>
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<td>1,2,3</td>
<td>9. L a</td>
<td>7, 8 \to O</td>
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<td>10. \neg L a</td>
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<td>11. L a \land \neg L a</td>
<td>9, 10 &amp;I</td>
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<td>1,2</td>
<td>12. \neg (x)(\neg S x \to L x)</td>
<td>3 – 11 \neg I</td>
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10. (Optional) “Only people who aren’t rational are mentally ill. Everybody who thinks clearly is rational. All philosophers think clearly. Therefore, there aren’t mentally ill philosophers.”

\[(x)(M x \to \neg R x), \quad (x)(C x \to R x), \quad (x)(P x \to C x) \vdash \neg (\exists x)(M x \land P x)\]

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<td>1. (x)(M x \to \neg R x)</td>
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<td>P - want \neg (\exists x)(M x \land P x)</td>
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<td>4. (\exists x)(M x \land P x)</td>
<td>PP - want contradiction</td>
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<td>5. M a \land P a</td>
<td>4 EO</td>
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<td>1</td>
<td>6. M a \to \neg R a</td>
<td>1 UO</td>
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<td>7. C a \to R a</td>
<td>2 UO</td>
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<td>3</td>
<td>8. P a \to C a</td>
<td>3 UO</td>
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<td>7,10 MT</td>
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<td>3,4</td>
<td>13. C a</td>
<td>8, 12 \to O</td>
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<td>1,2,3,4</td>
<td>14. C a \land \neg C a</td>
<td>11, 13 &amp;I</td>
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<td>1,2,3</td>
<td>15. \neg (\exists x)(M x \land P x)</td>
<td>4 – 14 \neg I</td>
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11. (Optional) “Someone is either lying or a murderer. Everyone who lies has sinned. Everyone who commits murder has sinned. Only people who haven’t sinned are innocent. Therefore, not everyone is innocent.”
\((\exists x)(Lx \lor Mx), (x)(Lx \rightarrow Sx), (x)(Mx \rightarrow Sx), (x)(Ix \rightarrow \neg Sx) \vdash \neg(x)Ix\)

1. \((\exists x)(Lx \lor Mx)\)  P
2. \((x)(Lx \rightarrow Sx)\)  P
3. \((x)(Mx \rightarrow Sx)\)  P
4. \((x)(Ix \rightarrow \neg Sx)\)  P - want \(\neg(x)Ix\)
5. \((x)Ix\)  PP – want contradiction
6. \(La \lor Ma\)  1 EO
7. \(La \rightarrow Sa\)  2 UO
8. \(Ma \rightarrow Sa\)  3 UO
9. \(Ia \rightarrow \neg Sa\)  4 UO
10. \(Ia\)  5 UO
11. \(Sa\)  6, 7, 8 SD
12. \(\neg Sa\)  9, 10 \(\rightarrow\)O
13. \(Sa \& \neg Sa\)  11, 12 \&I
14. \(\neg(x)Ix\)  5 – 13 \~I

12. (Optional) “Only forgiven people go to Heaven. All people who go to Hell are damned. But some people are neither forgiven nor damned. Therefore, some people go neither to Heaven nor Hell.”

\((x)(Vx \rightarrow Fx), (x)(Lx \rightarrow Dx), (\exists x)(\neg Fx \& \neg Dx) \vdash (\exists x)(\neg Vx \& \neg Lx)\)

1. \((x)(Vx \rightarrow Fx)\)  P
2. \((x)(Lx \rightarrow Dx)\)  P
3. \((\exists x)(\neg Fx \& \neg Dx)\)  P - want \((\exists x)(\neg Vx \& \neg Lx)\)
4. \(\neg Fa \& \neg Da\)  3 EO
5. \(Va \rightarrow Fa\)  1 UO
6. \(La \rightarrow Da\)  2 UO
7. \(\neg Fa\)  4 \&O
8. \(\neg Va\)  5, 7 MT
9. \(\neg Da\)  4 \&O
10. \(\neg La\)  6, 9 MT
11. \(\neg Va \& \neg La\)  8, 10 \&I
12. \((\exists x)(\neg Vx \& \neg Lx)\)  11 EI

13. (Optional) “Lies are statements that both false and known by the speaker to be false. Some false statements aren’t known by the speaker to be false. Therefore, not all false statements are lies.”

\((x)(Lx \rightarrow (Fx \& Kx)), (\exists x)(Fx \& \neg Kx) \vdash \neg(x)(Fx \rightarrow Lx)\)
Figuring out UI

So much for EO. Now, what about UI? It seems like UI would look something like this:

UI (Universal Quantifier In): From a line of the form \( Pn \), where “n” is any name, we may derive a line of the form \( (x)Px \).

Now this is a truly horrible rule because it allows us to say that if one particular thing has a certain property, then everything has a certain property! In particular, it would allow us to prove the following invalid arguments:

“George Washington was the first president of the United States. Therefore everyone was the first president of the United States.”

\[ Pw \vdash (x)Px \]

1. \( Pw \)  P - True
2. \( (x)Px \)  1 UI - False!

“Some students are bartenders. Therefore everyone is a bartender.”

\[ (\exists x)(Sx & Bx) \vdash (x)Bx \]

1. \( (\exists x)(Sx & Bx) \)  P - True
2. \( Sa & Ba \)  1 EO
3. \( Ba \)  2 &O
4. \( (x)Bx \)  3 UI - False!

The problem is that we can’t go from a specific claim, like ‘Pn,’ to a universal claim, like ‘(x)Px,’ when ‘n’ refers to a specific person. And there are two ways that a name can refer to
somebody in particular:

First, it can be somebody’s name, like ‘w’ referred to George Washington in argument 1.

Second, it can refer to a particular, unnamed, individual whom we know must exist in virtue of there being an existential claim about him, like ‘a’ referred to whomever is both a student and a bartender in argument 2.

This means that we can’t do UI over a name that appears in the symbolized argument or appeared previously in the proof as a result of EO. But there are only three ways that a name can occur in a proof: 1) it can occur as a name in the symbolized argument, 2) it can show up as a result of EO, or 3) it can show up as a result of UO. Thus, we can do UI only over names that appeared as a result of UO.

**UI (Universal Quantifier In):** From a line of the form Pn, where “n” was introduced as a result of UO, we may derive a line of the form (x)Px.

It helps to mark names that are introduced as the result of UO, in order to know that we may do UI on them later. Let’s use an asterisk for this mark, so “a” would be a regular name or would be introduced as a result of applying EO, but “a*” would be introduced as a result of applying UO.

Symbolize and prove:

1. “All historians are familiar with the Peloponnesian War. Everybody familiar with the Peloponnesian War can dominate a conversation. Therefore all historians can dominate a conversation.”

   \[(x)(Hx \rightarrow Fx), (x)(Fx \rightarrow Dx) \vdash (x)(Hx \rightarrow Dx)\]

   1. \((x)(Hx \rightarrow Fx)\) P
   2. \((x)(Fx \rightarrow Dx)\) P – want \((x)(Hx \rightarrow Dx)\)
   3. Ha* \rightarrow Fa* 1 UO
   4. Fa* \rightarrow Da* 2 UO
   5. Ha* \rightarrow Da* 3,4 CH
   6. \((x)(Hx \rightarrow Dx)\) 5 UI

2. (Optional) “All theists believe in a nonphysical God. Anyone who believes in a nonphysical God is a substance dualist. Therefore, all theists are substance dualists.”

   \[(x)(Tx \rightarrow Bx), (x)(Bx \rightarrow Dx) \vdash (x)(Tx \rightarrow Dx)\]

   1. \((x)(Tx \rightarrow Bx)\) P
   2. \((x)(Bx \rightarrow Dx)\) P - want \((x)(Tx \rightarrow Dx)\)
3. Ta* → Ba* 1UO
4. Ba* → Da* 2 UO
5. Ta* → Da* 3,4 CH
6. (x)(Tx → Dx) 5 UI

3. “Only people who believe in ghosts are substance dualists. Nobody who isn’t superstitious believes in ghosts. Therefore, only superstitious people are substance dualists.”

(x)(Dx → Bx), (x)(~Sx → ~Bx) |- (x)(Dx → Sx)
1. (x)(Dx → Bx) P
2. (x)(~Sx → ~Bx) P - want (x)(Dx → Sx)
3. Da* → Ba* 1 UO
4. ~Sa* → ~Ba* 2 UO
5. Ba* → Sa* 4 CN
6. Da* → Sa* 3, 5 CH
7. (x)(Dx → Sx) 6 UI

4. (Optional) Prove that “All lawyers are shrewd” is logically equivalent to “Nobody who isn’t shrewd is a lawyer.”

(x)(Lx → Sx) ≡ (x)(~Sx → ~Lx)
1. (x)(Lx → Sx) P
2. (x)(~Sx → ~Lx) 1 CN

5. (Optional) Prove that “All philosophers are obscure” is logically equivalent to “Everyone is either obscure or not a philosopher.”

(x)(Px → Ox) ≡ (x)(Ox v ~Px)
1. (x)(Px → Ox) P
2. (x)(~Px v Ox) 1 AR
3. (x)(Ox v ~Px) 2 v COM

1. (x)(Ox v ~Px) P
2. (x)(~Px v Ox) 1 AR
3. (x)(Px → Ox) 2 v COM
There is only one more inference rule for us to learn!

**QE (Quantifier Exchange):**
- From a line of the form \(\ldots \lnot(x)Px\ldots\) we may derive a line of the form \(\ldots (\exists x)\lnot Px\ldots\), and vice versa.
- From a line of the form \(\ldots \lnot(\exists x)Px\ldots\) we may derive a line of the form \(\ldots (x)\lnot Px\ldots\), and vice versa.

QE is valid because if it isn’t the case that all things have a property then something doesn’t have that property, and if it isn’t the case that something has a property then everything fails to have that property.

In fact, quantifier exchange is actually a contraction of DeMorgan’s Law.

Suppose that Alice, Bob and Carol are the only people in the world and we’re discussing how many of them, if any, are philosophers.

If we want to say that all of them are philosophers, we’d write:
\[(x)Px = Pa \land Pb \land Pc\]

If we want to say that some of them are philosophers, we’d write:
\[(\exists x)Px = Pa \lor Pb \lor Pc\]

See the following equivalences, using DeMorgan’s?
\[\lnot(x)Px = \lnot(Pa \land Pb \land Pc) = \lnot Pa \lor \lnot Pb \lor \lnot Pc = (\exists x)\lnot Px\]
\[\lnot(\exists x)Px = \lnot(Pa \lor Pb \lor Pc) = \lnot Pa \land \lnot Pb \land \lnot Pc = (x)\lnot Px\]

QE is needed because it transforms negations into universal or existential quantifications, which we can then manipulate using UO or EO. As such, it’s always a good idea to do QE right away. That way you’ll know what universal quantifications and what existential quantifications you have, which will enable you to do EO before UO. So, do QE first, then do EO, and then do UO.

Symbolize and prove:

1. “All poodles are adorable animals. It is not the case that all poodles can perform tricks. Therefore, some adorable animals can’t perform tricks.”

\[(x)(Px \to Ax), \lnot(x)(Px \to Tx) \vdash (\exists x)(Ax \land \lnot Tx)\]

\[\begin{align*}
1. & \quad (x)(Px \to Ax) & \quad P \\
2. & \quad \lnot(x)(Px \to Tx) & \quad P \quad \text{– want } (\exists x)(Ax \land \lnot Tx) \\
3. & \quad (\exists x)\lnot(Px \to Tx) & \quad 2 \text{ QE} \\
4. & \quad \lnot(Pa \to Ta) & \quad 3 \text{ EO}
\end{align*}\]
5. \( Pa \rightarrow Aa \)  
6. \( \neg(\neg Pa \lor Ta) \)  
7. \( \neg\neg Pa \land \neg Ta \)  
8. \( Pa \land \neg TA \)  
9. \( Pa \)  
10. \( Aa \)  
11. \( \neg Ta \)  
12. \( Aa \land \neg Ta \)  
13. \( \exists x)(Ax \land \negTx) \)

2. “Everything that speaks Spanish knows what ‘Hola’ means. It is not the case that some cats know what ‘Hola’ means. Therefore, it is not the case that some cats speak Spanish.”

\[(x)(Sx \rightarrow Kx), \neg(\exists x)(Cx \land Kx) \vdash \neg(\exists x)(Cx \land Sx)\]

1. \( (x)(Sx \rightarrow Kx) \)  
2. \( \neg(\exists x)(Cx \land Kx) \)  
3. \( \exists x)(Cx \land Sx) \) – want contradiction

2. 4. \( (x)\neg(Cx \land Kx) \)  
5. \( Ca \land Sa \)  
1 6. \( Sa \rightarrow Ka \)  
2 7. \( \neg(Ca \land Ka) \)  
etc.

3. (Optional) “Objective judgments are either true independently of people or else are scientifically provable. No value judgments are true independently of people, and no value judgments are scientifically provable. Therefore, no objective value judgments exist.

\[(x)(Ox \rightarrow (Ix \lor Px)), \neg(\exists x)Ix \land \neg(\exists x)Px \vdash \neg(\exists x)Ox\]

1. \( (x)(Ox \rightarrow (Ix \lor Px)) \)  
2. \( \neg(\exists x)Ix \land \neg(\exists x)Px \)  
3. \( (\exists x)Ix \)  
4. \( (\exists x)Px \)  
5. \( (x)\neg Ix \)  
6. \( (x)\neg Px \)  
7. \( Oa^* \rightarrow (Ia^* \lor Pa^*) \)  
8. \( \neg Ia^* \)  
9. \( \neg Pa^* \)  
10. \( \neg Ia^* \land \neg Pa^* \)  
11. \( (Ia^* \lor Pa^*) \)  
12. \( \neg Oa^* \)  

etc.
13. (x)~Ox  12 UI
14. ~(∃x)Ox  13 QE

4. (Optional) “No mental events are physical things. Therefore, no mental events studied by neurophysiology are physical things.”

1 1. (x)(Mx → ~Px) P - want (x)((Mx & Sx) → ~Px)
2 2. ~((x)((Mx & Sx) → ~Px)) PP - want contradiction
2 3. (∃x)~((Mx & Sx) → ~Px) 2 QE
2 4. ~((Ma & Sa) → ~Pa) 3 EO
2 5. ~(~(Ma & Sa) ∨ ~Pa) 4 AR
2 6. ~((Ma & Sa) & ~~Pa) 5 DM
2 7. Ma & Sa & Pa 6 DN
1 8. Ma → ~Pa 1 UO
2 9. Ma 7 &O
1,2 10. ~Pa 8,9 →O
2 11. Pa 7 &O
1,2 12. Pa & ~Pa 10, 11 &I
1 13. (x)((Mx & Sx) → ~Px) 2-12 ~I

5. (Optional) (∃x)(Ax & Bx), (x)(Bx → ~Cx), (x)(Dx → ~Ax) |- ~(x)(~Cx → Dx)

1. (∃x)(Ax & Bx) P
2. (x)(Bx → ~Cx) P
3. (x)(Dx → ~Ax) P - want ~(x)(~Cx → Dx)
   = (∃x)~(~Cx→Dx)
   = (∃x)(~Cx & ~Dx)
4. Aa & Ba 1 EO
5. Ba → ~Ca 2 UO
6. Da → ~Aa 3 UO
7. Aa 4 &O
8. Ba 4 &O
9. ~Ca 5, 8 →O
10. ~Da 6, 7 MT
11. ~Ca & ~Da 9, 10 &I
12. (∃x)(~Cx & ~Dx) 11 EI
13. (∃x)~(~Cx→Dx) 12 Neg AR
14. ~(x)(~Cx → Dx) 13. QE

Quiz time.
Universe of Discourse (Optional)

What we’ve just learned is sufficient for symbolizing and proving arguments in a very large segment of predicate logic. But there is a way to make the symbolizations a little simpler.

The universe of discourse (UD) is the set of things we’re quantifying over. Unless otherwise stated, the UD will be everything, but if we restrict our universe of discourse, we can simplify the symbolization.

“Some philosophers are obstreperous atheists.” \( (\exists x)(Px \land Ox \land Ax) \)
UD: Philosophers \( (\exists x)(Px \land Ax) \)
UD: Obstreperous philosophers \( (\exists x)(Ax) \)

“Obsequious theologians are seldom invited to dinner parties.” \( (x)((Ox \land Tx) \rightarrow Sx) \)
UD: Theologians \( (x)(Ox \rightarrow Sx) \)
UD: Obsequious theologians \( (x)(Sx) \)

“Philosophy articles that employ symbolic logic are seldom about God unless they’re by Alvin Plantiga.” \( (x)((Px \land Sx) \rightarrow (\neg Ax \rightarrow \neg Gx)) \)
UD: Philosophy articles \( (x)(Sx \rightarrow (\neg Ax \rightarrow \neg Gx)) \)
UD: Philosophy articles that employ symbolic logic. \( (x)(\neg Ax \rightarrow \neg Gx) \)

Symbolize and prove:

1. “Only nonphysical mental events could be completely private. But there are no nonphysical mental events and so no mental events are completely private.”
UD: mental events.

\[
\begin{align*}
1. \ & (x)(Rx \rightarrow \neg Px) & P \\
2. \ & \neg(\exists x)\neg Px & P - \text{want } \neg(\exists x)(Rx) \\
3. \ & (\exists x)Rx & PP - \text{want contradiction} \\
4. \ & (x)\neg\neg Px & 2 \text{ QE} \\
5. \ & Ra & 3 \text{ EO} \\
6. \ & Ra \rightarrow \neg Pa & 1 \text{ UO} \\
7. \ & \neg Pa & 6,7 \rightarrow O \\
8. \ & \neg\neg Pa & 4 \text{ UO} \\
9. \ & \neg Pa \land \neg Pa & 7,8 \& I \\
10. \ & \neg(\exists x)(Rx) & 3-9 \neg I \\
\end{align*}
\]

2. “If dualism is right then Ryle is wrong. If dualism isn’t right then Descartes is wrong. Thus, either way, some philosopher is wrong.”
UD: Philosophers
1. \( \text{Rd} \to \text{Wr} \)
2. \( \neg\text{Rd} \to \text{Wg} \)
3. \( \neg(\exists x)\text{Wx} \)
4. \( (x)\neg\text{Wx} \)
5. \( \neg\text{Wr} \)
6. \( \neg\text{Wr} \to \neg\text{Rd} \)
7. \( \neg\text{Rd} \)
8. \( \neg\text{Wg} \)
9. \( \neg\text{Wg} \to \neg\text{Rd} \)
10. \( \neg\text{Rd} \)
11. \( \neg\text{Rd} \& \neg\text{Rd} \)
12. \( (\exists x)\text{Wx} \)

3. “All dualist theologians are mistaken. Therefore if everyone is a dualist theologian then everyone is mistaken.”

UD: Theologians

<table>
<thead>
<tr>
<th>Step</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( (x)\text{Dx} \to \text{Mx} )</td>
</tr>
<tr>
<td>2</td>
<td>( (x)\text{Dx} )</td>
</tr>
<tr>
<td>3</td>
<td>( \neg(x)\text{Mx} )</td>
</tr>
<tr>
<td>4</td>
<td>( (\exists x)\neg\text{Mx} )</td>
</tr>
<tr>
<td>5</td>
<td>( \neg\text{Ma} )</td>
</tr>
<tr>
<td>6</td>
<td>( \text{Da} \to \text{Ma} )</td>
</tr>
<tr>
<td>7</td>
<td>( \text{Da} )</td>
</tr>
<tr>
<td>8</td>
<td>( \text{Ma} )</td>
</tr>
<tr>
<td>9</td>
<td>( \text{Ma} &amp; \neg\text{Ma} )</td>
</tr>
<tr>
<td>10</td>
<td>( \neg\text{Dx} )</td>
</tr>
<tr>
<td>11</td>
<td>( \text{Da} \to \text{Aa} )</td>
</tr>
<tr>
<td>12</td>
<td>( \neg\text{Da} )</td>
</tr>
</tbody>
</table>

4. “If some philosopher is a dualist then all philosophers are ashamed. Therefore all dualist philosophers are ashamed.”

UD: Philosophers

<table>
<thead>
<tr>
<th>Step</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( (\exists x)\text{Dx} \to (x)\text{Ax} )</td>
</tr>
<tr>
<td>2</td>
<td>( (x)(\text{Dx} \to \text{Ax}) )</td>
</tr>
<tr>
<td>3</td>
<td>( \neg(x)(\text{Dx} \to \text{Ax}) )</td>
</tr>
<tr>
<td>4</td>
<td>( (\exists x)\neg\text{Dx} )</td>
</tr>
<tr>
<td>5</td>
<td>( (x)\neg\text{Dx} )</td>
</tr>
<tr>
<td>6</td>
<td>( \neg\text{Da} \to \text{Aa} )</td>
</tr>
<tr>
<td>7</td>
<td>( \neg\text{Da} )</td>
</tr>
<tr>
<td>8</td>
<td>( \neg\neg\text{Da} )</td>
</tr>
<tr>
<td>9</td>
<td>( \neg\text{Da} )</td>
</tr>
</tbody>
</table>
5. “Either all mental events are intentional or no mental events can be simulated on a computer. Some mental events are intentional. All intentional events can be simulated on a computer. Therefore all mental events are intentional.”

UD: Mental events

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1. (x)Ix ∨ (x)¬Sx</td>
<td>P</td>
<td></td>
</tr>
<tr>
<td>2. (∃x)Ix</td>
<td>P</td>
<td></td>
</tr>
<tr>
<td>3. (x)(Ix → Sx)</td>
<td>P - want (x)Ix, want ¬(x)¬Sx</td>
<td></td>
</tr>
<tr>
<td>4. (x)¬Sx</td>
<td>PP - want contradiction</td>
<td></td>
</tr>
<tr>
<td>5. Ia</td>
<td>2 EO</td>
<td></td>
</tr>
<tr>
<td>6. Ia → Sa</td>
<td>3 UO</td>
<td></td>
</tr>
<tr>
<td>7. Sa</td>
<td>5,6 →O</td>
<td></td>
</tr>
<tr>
<td>8. ¬Sa</td>
<td>4 UO</td>
<td></td>
</tr>
<tr>
<td>9. Sa &amp; ¬Sa</td>
<td>7,8 &amp;I</td>
<td></td>
</tr>
<tr>
<td>10. ¬(x)¬Sx</td>
<td>4-9 ¬I</td>
<td></td>
</tr>
<tr>
<td>11. (x)Ix</td>
<td>1,10 vO</td>
<td></td>
</tr>
</tbody>
</table>

Diagrams (Optional)

Diagrams serve much the same role in predicate logic that truth tables serve in propositional logic: they help us to see the meaning of sentences and they help us to determine both the validity and the invalidity of arguments. We’ll start by seeing how to diagram sentences.

Diagramming Sentences

Circles will represent predicates. We’ll think of everything that falls under the predicate as included in the circle.

A name (‘a,’ ‘b,’ ‘c,’ etc.) represents a specific, identified individual. A variable (‘x’, ‘y’, or ‘z’) represents an unknown individual.

Take a look at the following three diagrams. Can you see how they represent the sentences?
“George is fun at parties, but not a philosopher.”

```
\begin{center}
\begin{tikzpicture}
\fill[red!30!white] (0,0) rectangle (3,3);
\fill[blue!30!white] (3,0) rectangle (6,3);
\draw (0,0) -- (6,0);
\draw (0,3) -- (6,3);
\draw (0,0) ellipse (1 and 1);
\draw (3,0) ellipse (1 and 1);
\node at (1.5,1.5) {g};
\node at (1.5,1.5) {P};
\node at (4.5,1.5) {F};
\end{tikzpicture}
\end{center}
```

“Some philosophers are fun at parties.”

```
\begin{center}
\begin{tikzpicture}
\fill[red!30!white] (0,0) rectangle (3,3);
\fill[blue!30!white] (3,0) rectangle (6,3);
\draw (0,0) -- (6,0);
\draw (0,3) -- (6,3);
\draw (0,0) ellipse (1 and 1);
\draw (3,0) ellipse (1 and 1);
\node at (1.5,1.5) {x};
\node at (1.5,1.5) {P};
\node at (4.5,1.5) {F};
\end{tikzpicture}
\end{center}
```

“Some philosophers are not fun at parties.”

```
\begin{center}
\begin{tikzpicture}
\fill[red!30!white] (0,0) rectangle (3,3);
\fill[blue!30!white] (3,0) rectangle (6,3);
\draw (0,0) -- (6,0);
\draw (0,3) -- (6,3);
\draw (0,0) ellipse (1 and 1);
\draw (3,0) ellipse (1 and 1);
\node at (1.5,1.5) {x};
\node at (1.5,1.5) {P};
\node at (4.5,1.5) {F};
\end{tikzpicture}
\end{center}
```

If we want to say that nothing occupies a certain part of the diagram, we’ll shade it out. Think about the following two diagrams. See how it works? For “All philosophers are fun at parties,” I shaded out the segment that corresponds to philosophers who aren’t fun at parties, thereby “forcing” all philosophers into the “fun at parties” circle. For “No philosophers are fun at parties,” I shaded in the segment of the diagram where philosophers who are fun at parties would be, signaling that nothing is there and that (sadly) no philosophers are fun at parties.

```
\begin{center}
\begin{tikzpicture}
\fill[red!30!white] (0,0) rectangle (3,3);
\fill[blue!30!white] (3,0) rectangle (6,3);
\draw (0,0) -- (6,0);
\draw (0,3) -- (6,3);
\draw (0,0) ellipse (1 and 1);
\draw (3,0) ellipse (1 and 1);
\end{tikzpicture}
\end{center}
```

“All philosophers are fun at parties.”

```
\begin{center}
\begin{tikzpicture}
\fill[red!30!white] (0,0) rectangle (3,3);
\fill[blue!30!white] (3,0) rectangle (6,3);
\draw (0,0) -- (6,0);
\draw (0,3) -- (6,3);
\draw (0,0) ellipse (1 and 1);
\draw (3,0) ellipse (1 and 1);
\end{tikzpicture}
\end{center}
```

“No philosophers are fun at parties.”

```
\begin{center}
\begin{tikzpicture}
\fill[red!30!white] (0,0) rectangle (3,3);
\fill[blue!30!white] (3,0) rectangle (6,3);
\draw (0,0) -- (6,0);
\draw (0,3) -- (6,3);
\draw (0,0) ellipse (1 and 1);
\draw (3,0) ellipse (1 and 1);
\end{tikzpicture}
\end{center}
```

When we have more than two predicates, things can get more interesting but the same principles apply. Take a look at the diagrams for the following sentences and study them until they make perfect sense.
“Some British philosophers aren’t fun at parties.”

“All British philosophers are fun at parties.”

“No British philosophers are fun at parties.”

“All philosophers are British and fun at parties.”

If we want to say that something has one or more properties, we put a variable corresponding to that thing in each of the relevant segments and connect the variables with lines signifying that we’re not sure exactly where it might be sitting among the possibilities represented by the segments. Examine the following sentence and diagram to see how this works.

“Some philosophers are British or fun at parties (or both).”
Diagramming Arguments

Now that we know how to diagram sentences, we can use diagrams to assess arguments for validity by diagramming all of the argument’s premises on the same diagram and seeing if the conclusion also ends up being represented.

An argument is valid iff a diagram that represents the premises must also represent the conclusion. An argument is invalid iff a diagram can represent the premises and the opposite of the conclusion.

(Tip: It helps to diagram universal premises before the existential ones.)

Take a look at how we can assess the following arguments by diagramming them.

1. “All philosophers are fun at parties. Jennifer is a philosopher. Thus, Jennifer is fun at parties.”

2. “All philosophers are fun at parties. Jennifer is fun at parties. Thus, Jennifer is a philosopher.”

<table>
<thead>
<tr>
<th></th>
<th>P</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Valid

Invalid
3. “All philosophers are fun at parties. Leo isn’t fun at parties. Thus, Leo isn’t a philosopher.”

Valid

4. “All philosophers are fun at parties. Leo isn’t a philosopher. Thus, Leo isn’t fun at parties.”

Invalid

5. “Sam is a philosopher. Sam isn’t fun at parties. Thus, it’s not the case that all philosophers are fun at parties.” Note: “It’s not the case that all philosophers are fun at parties” = “Some philosophers aren’t fun at parties.”

Valid
6. “Sam is a philosopher. Sam isn’t fun at parties. Thus, no philosophers are fun at parties.”

   ![Diagram for 6.]

   Invalid

7. “Ellen is either a philosopher or a theologian. All philosophers are interested in the idea of God. All theologians are interested in the idea of God. Thus Ellen is interested in the idea of God.”

   ![Diagram for 7.]

   Valid
8. “Ellen is either a philosopher or a theologian. No philosophers are interested in the idea of God. All theologians are interested the idea of God. Thus, Ellen is interested in the idea of God.”

```
P  e
T
```

Invalid

9. “Some philosophers are theists. No theists are convinced by the problem of evil. Therefore, not all philosophers are convinced by the problem of evil.”

```
P  x
T
```

Valid

10. “Some philosophers are theists. Some theists aren’t convinced by the problem of evil. Therefore, not all philosophers are convinced by the problem of evil.”

```
P  y
T
```

Invalid (I diagramed the opposite of the conclusion and both premises)
11. “All rationalists are adept at logic. But nobody adept at logic would be misled by the fallacy of assuming the consequent. Therefore, it’s false that there are rationalists who are misled by the fallacy of assuming the consequent.”

![Valid Diagram]

12. “Some rationalists are adept at logic. But nobody adept at logic would be misled by the fallacy of assuming the consequent. Therefore, it’s false that there are rationalists who are misled by the fallacy of assuming the consequent.”

![Invalid Diagram]
13. “All theists believe in a nonphysical God. Anyone who believes in a nonphysical God is a substance dualist. Therefore, all theists are substance dualists.”

![Diagram for 13](image)

Valid

14. “All theists believe in a nonphysical God. Anyone who is a substance dualist believes in a nonphysical God. Therefore, all theists are substance dualists.”

![Diagram for 14](image)

Invalid

15. “Some atheists believe in reincarnation. Anyone who believes in reincarnation is a substance dualist. Therefore, some atheists are substance dualists.”

![Diagram for 15](image)

Valid
16. “Some atheists believe in reincarnation. Anyone who is a substance dualist believes in reincarnation. Therefore, some atheists are substance dualists.”

![Venn Diagram](Image)

Invalid (I diagramed the opposite of the conclusion and both premises)

### Establishing Invalidity by the Method of Interpretations (Optional)

Proofs establish validity; if we can prove an argument then it’s valid. Proofs don’t establish invalidity, however: if we can’t prove an argument, we can’t conclude that the argument is invalid. This means that we need a way to establish that an argument is invalid. We could do this by means of the diagram method discussed above, but diagrams get very complex and unwieldy if the argument has more than three predicates. The following is a technique is another way to establish that a predicate argument is invalid.

<table>
<thead>
<tr>
<th>The Method of Counterexamples</th>
</tr>
</thead>
<tbody>
<tr>
<td>We often prove that an argument is invalid by constructing a counterexample - another argument of the same form with true premises and a false conclusion:</td>
</tr>
</tbody>
</table>

For example, “All *a priori* truths are necessary. ‘2 + 2 = 4’ is a necessary truth. Therefore, ‘2 + 2 = 4’ is *a priori.*” has the same form as “All women are human. Ralph is human. Therefore, Ralph is a woman.”

The second argument has true premises and a false conclusion and so is clearly invalid. But validity is a property of an argument’s form, so any argument of that form must be invalid. The first argument is of that form, so the first argument is invalid, too.

<table>
<thead>
<tr>
<th>The Method of Interpretations</th>
</tr>
</thead>
<tbody>
<tr>
<td>The method of interpretations is just a systematization and formalization of the method of counterexamples. Here’s how to do it:</td>
</tr>
</tbody>
</table>

---

2 Pospesel, *Predicate Logic*, Chapter 7
1. Symbolize the argument to be evaluated.
2. Try to find an interpretation of the predicate letters and names that make all of the premises true and the conclusion false. You may assign the same property to different predicate letters and you may assign the same individual to different names. Sometimes it helps to start with an interpretation that makes the conclusion false and then to complete that interpretation in a way that makes the premises true.
3. An argument is valid if and only if every interpretation that makes all of the premises true also makes the conclusion true, or equivalently, if and only if there is no interpretation which makes all the premises true and the conclusion false. Thus, if you can find an interpretation that makes all of the premises true and the conclusion false, the original argument is invalid.

Example 1

“All a priori truths are necessary. ‘2 + 2 = 4’ is a necessary truth. Therefore, ‘2 + 2 = 4’ is a priori.”

\((x)(Ax \rightarrow Nx), Nt \vdash At\)

Let
\(Ax = x \text{ is a woman.}\)
\(Nx = x \text{ is human.}\)
\(t = \text{Ralph}\)

“All women are human,” is true.
“Ralph is human,” is true.
“Ralph is a woman,” is false.
So this argument is invalid.

Example 2

“Some ethical theories don’t involve assessing consequences. Some ethical theories support the rights of animals. Therefore, some theories which don’t involve assessing consequences support the rights of animals.”

\((\exists x)(Ex \& \neg Cx), (\exists x)(Ex \& Rx) \vdash (\exists x)(\neg Cx \& Rx)\)

Let
\(Ex = x \text{ is a physics major.}\)
\(Cx = x \text{ is a liberal arts major}\)
\(Rx = x \text{ is a philosophy major.}\)

“Some physics majors aren’t liberal arts majors,” is true.
“Some physics majors are philosophy majors,” is true.
“Some philosophy majors aren’t liberal arts majors,” is false.
So this argument is invalid.
Example 3

“Any argument for God’s existence that appeals to evidence from nature is invalid. No ontological argument appeals to evidence from nature. Therefore, all arguments for God’s existence which are not ontological are invalid.”

UD: Arguments

\[(x)[(Gx \land Nx) \rightarrow \neg Vx], (x)(Ox \rightarrow \neg Nx) \vdash (x)[(Gx \land \neg Ox) \rightarrow \neg Vx]\]

Let

UD = people

Gx = x is over 5 feet tall.

Nx = x is male.

Vx = x is female.

Ox = x is a mother.

“No men over 5 feet tall are female,” is true.

“No mothers are male,” is true.

“No people who are both over 5 feet tall and not mothers are female,” is false.

So this argument is invalid.

Example 4

“All philosophers understand logic. Therefore, some philosophers understand logic.”

\[(x)(Px \rightarrow Ux) \vdash (\exists x)(Px \land Ux)\]

Let

Px = x is a unicorn.

Ux = x has one horn.

“All unicorns have one horn,” is true (because it’s equivalent to “There are no unicorns which don’t have one horn,” which is true because there aren’t any unicorns at all.”)

“There is a unicorn with one horn,” is false.

Also, since we may assign the same property to different predicate letters and we may assign the same individual to the same name, the following interpretation will also work:

Let

Px = x is a unicorn.

Ux = x is a unicorn.
“All unicorns are unicorns,” is true (because it’s equivalent to “There are no unicorns which aren’t unicorns,” which is true because there aren’t any unicorns at all.”). “There is something which is a unicorn and a unicorn,” is false. So this argument is invalid.

Example 5

“If all philosophers are dualists then all philosophers are mistaken. Therefore, all dualist philosophers are mistaken.”

UD: philosophers
(x) Dx → (x) Mx |- (x) (Dx → Mx)

Let
UD: {1,2,3,4,5}
Dx = x is divisible by 2.
Mx = x is a multiple of 4.

“If all numbers in the UD are divisible by 2 then all numbers in the UD are multiples of 4,” is true because the antecedent is false.
“All numbers in the UD that are divisible by 2 are multiples of 4,” is false because 2 is divisible by 2 yet not a multiple of 4.
So this argument is invalid.

Example 6

“Anything that’s governed by strict rules of behavior is a machine. But human beings aren’t governed by strict rules of behavior and so human beings aren’t machines.”

(x) (Gx → Mx), (x) (Hx → ~Gx) |- (x) (Hx → ~Mx)

Let
Gx = x is a fish.
Mx = x can swim.
Hx = x is human.

“All fish can swim,” is true.
“No humans are fish,” is true.
“No humans can swim,” is false.
So this argument is invalid.