Four Ways to Represent a Function

Section 1.1

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Definitions

A function $f$ is a rule that assigns to each element $x$ in a set $A$ exactly one element, called $f(x)$, in a set $B$.

$A$ is the domain of the function $f$

Independent variable is the variable representing a number in the domain.

$B$ is the codomain of the function $f$ (Not in book)

$f(x)$ is the value of $f$ at $x$.

The range of $f$ is the set of all values of $f$.

Dependent variable is the variable representing a number in the range.

Domains and ranges are usually given in interval notation (See Appendix A)
Visualizing Functions

- Machine diagram

- Arrow diagram

- Graph
Representations of Functions

- Verbally
  - The height above the ground of parachutist as a function of time,
  - The value of a computer over time,
  - The number of minutes it takes water to come to a boil as a function of altitude,
  - The efficiency (miles per gallon) of an auto with respect to speed (miles per hour).

- Numerically
  - A table of values

- Visually
  - Usually a graph

- Algebraically
  - A formula
Theorem (The Vertical Line Test)

A curve in the $xy$-plane is the graph of a function of $x$ if and only if no vertical line intersects the curve more than once.
Piecewise Defined Functions

- \( f(x) = \begin{cases} 
2x & \text{if } 0 \leq x < 2 \\
3x^2 & \text{if } 2 \leq x \\
-1 & \text{if } x < -1 
\end{cases} \)

- \( f(x) = \begin{cases} 
3x + 2 & \text{if } |x| < 1 \\
7 - 2x & \text{if } x > 1 
\end{cases} \)

- **Absolute value function**
  - \(|a| = \begin{cases} 
a & \text{if } a \geq 0 \\
-a & \text{if } a < 0 
\end{cases} \)
Piecewise Defined Functions

- **Step functions**
  - \([x]\) is the *greatest integer* function, the largest integer less than or equal to \(x\).
  - \([x]\) is the *least integer* function, the smallest integer greater than or equal to \(x\).

- **Heaviside Function**: \(H(x) = \begin{cases} 0 & \text{if } x < 0 \\ 1 & \text{if } x \geq 1 \end{cases} \)
Symmetry

Definition

- If \( f(-x) = f(x) \), then \( f \) is an **even function**
- If \( f(-x) = -f(x) \), then \( f \) is an **odd function**

An even function is symmetric with respect to the \( y \)-axis.

An odd function is symmetric about the origin.
Symmetry

Examples:

- $f(x) = e^{x^2 - 1}$
- $g(x) = x(x^2 - 1)$
- $h(x) = x^3 + 1$
- $j(x) = \frac{x}{x^3 + x}$
Increasing and Decreasing Functions

Definition

- Definition. A function $f$ is called **increasing** on an interval $I$ if $f(x_1) < f(x_2)$ whenever $x_1 < x_2$ in $I$.
- Definition. A function $f$ is called **decreasing** on an interval $I$ if $f(x_1) > f(x_2)$ whenever $x_1 > x_2$ in $I$. 

![Diagram of Increasing and Decreasing Functions](image-url)