Leontief Input-Output Analysis

Section 4-7

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1 Input-Output Analysis

1.1 Input-Output Analysis

Input-Output Analysis

- Determine relations between industries

- For example:
  - Electric company requires water to produce electricity
  - Water company requires electricity to produce water
  - How much total water/electricity is needed to meet outside demands?

- Wassily Leontief was awarded Nobel Prize in 1973 for work on this question

- Work dealt with 500 industries - we will use at most three

1.2 Two Industry Model

Setup of Problem

- Idea: Total output for an industry \((x_i)\) depends on
  - Demand by other industries (\(\text{Internal demand}\))
  - Demand by outside (\(\text{Final demand}\))

- Three matrices:
  - \(\text{Final demand matrix} \ D \ (2 \times 1 \text{ matrix})\)
  - \(\text{Output matrix} \ X \ (2 \times 1 \text{ matrix})\)
  - \(\text{Technology matrix} \ M \ (2 \times 2 \text{ matrix})\)
  - Technology matrix holds information giving requirements of related industries

- Internal demand is \(MX\)

- Total demand (output) is internal demand and final demand together

- \(X = MX + D\)
Setup of Problem

- Technology matrix holds most data
- Label rows and columns by industries
- Must use same order for rows as for columns
- Rows represent input; Columns output

Example. An economy is based on Energy (E) and Transportation (T). In order to produce a dollar’s worth of output from the energy sector, input of $0.20 from energy and $0.35 from transportation is needed. For a dollar’s worth of output from the transportation sector, input of $0.30 from energy and $0.15 of transportation is needed. The technology matrix describing this economy is

\[
\begin{bmatrix}
0.20 & 0.30 \\
0.35 & 0.15
\end{bmatrix} = M
\]

Solution

- Solve \( X = MX + D \)
- Solution is \( X = (I - M)^{-1}D \) provided \( I - M \) has an inverse

Example

Problem 1. The technology matrix for a two-industry economy (agriculture and manufacturing) is

\[
\begin{bmatrix}
0.20 & 0.60 \\
0.40 & 0.20
\end{bmatrix} = M
\]

The final demand matrix is \( D = \begin{bmatrix} 800 \\ 1000 \end{bmatrix} \)

A. \( X = \begin{bmatrix} -1400 \\ 400 \end{bmatrix} \)
B. \( X = \begin{bmatrix} 40 \\ 480 \end{bmatrix} \)
C. \( X = \begin{bmatrix} 2200 \\ 600 \end{bmatrix} \)
D. \( X = \begin{bmatrix} 3100 \\ 2800 \end{bmatrix} \)
E. None of the above
Example

Problem 2. An economy has only two industries: the electric company and the gas company. Production of a dollar’s worth of electricity requires an input of $0.25 of electricity and $0.25 of gas. For the gas company, each dollar’s production requires $0.20 of gas and $0.40 electricity. What should the production of electricity and gas be (in dollars) if there is a $16 million demand for electricity and a $7 million demand for gas?

A. Electricity: $6.8 million; Gas: $5.4 million
B. Electricity: $24 million; Gas: $38 million
C. Electricity: $29.1 million; Gas: $23.3 million
D. Electricity: $31.2 million; Gas: $18.5 million
E. None of the above

1.3 Three Industry Model

Three Industry Model

- Works as in two industries
- Now sizes of matrices are:
  - Final demand $D$: $3 \times 1$
  - Output $X$: $3 \times 1$
  - Technology $M$: $3 \times 3$
- Solution is still $X = (I - M)^{-1}D$ assuming $I - M$ has an inverse

Three Industry Model

Example. An economy is based on Electricity (E), Natural Gas (N) and Oil (O). In order to produce a dollar’s worth of electricity, input of $0.30 of electricity, $0.10 of natural gas and $0.20 of oil are required. For a dollar’s worth of natural gas, inputs of $0.30 from electricity, $0.10 of natural gas and $0.20 from oil are necessary. For production of a dollar’s worth of oil, $0.10 is needed from each sector. The technology matrix is

$$
\begin{bmatrix}
E & N & O \\
E & 0.30 & 0.30 & 0.10 \\
N & 0.10 & 0.10 & 0.10 \\
O & 0.20 & 0.20 & 0.10
\end{bmatrix} = M
$$
Summary

You should be able to:

- Setup and solve the two industry Leontief Model
- Setup and solve the three industry Leontief Model