Review: Systems of Linear Equations in Two Variables

Section 4-1

Prof. Nathan Wodarz

Math 109 - Fall 2008

Contents

1 Systems in Two Variables 2
   1.1 Systems in Two Variables ................................. 2
   1.2 Graphing Systems of Equations ......................... 2

2 Solving Systems of Equations 4
   2.1 Terminology .............................................. 4
   2.2 Possible Solutions ...................................... 4
   2.3 Substitution .............................................. 5
   2.4 Elimination by Addition ................................. 6

3 Applications .............................................. 7
1 Systems in Two Variables

1.1 Systems in Two Variables

Systems in Two Variables

- More than one variable and more than one equation
- Each equation lets us possibly solve for a variable
- A linear system is
  \[ ax + by = h \]
  \[ cx + dy = k \]
- An ordered pair \((x_0, y_0)\) is a solution of the equation if the ordered pair satisfies both equations
- Set of all ordered pairs solving the system is the solution set
- To solve the equation is to find the solution set
- Three methods of solving
  - Graphing
  - Substitution
  - Elimination by addition

1.2 Graphing Systems of Equations

Graphing Systems of Equations

- Graph each line. Solution is where the lines intersect.
Graphing Systems of Equations

Problem 1. Solve the system of equations by graphing.

\[ \begin{align*}
  x - y &= -3 \\
  3x - y &= -1
\end{align*} \]

A. \((-1, -4)\)
B. \((1, 4)\)
C. \((4, 1)\)
D. No solution
E. Infinitely many solutions
F. None of the above
2 Solving Systems of Equations

2.1 Terminology

Terminology

- A system of linear equations is *consistent* if it has at least one solution
- A system is *inconsistent* if no solutions exist
- A consistent system is *independent* if its has exactly one solution (the *unique solution*)
- A consistent system is *dependent* if it has infinitely many solutions
- Why is there no terminology for more than one solution, but fewer than infinitely many?
- Two systems are *equivalent* if they have the same solution set

2.2 Possible Solutions

Possible Solutions

- A system of linear equations

\[
ax + by = h \\
cx + dy = k
\]

has

- Exactly one solution (consistent and independent)
- No solution (inconsistent)
- Infinitely many solutions (consistent and dependent)
- No other possibilities
2.3 Substitution

Substitution

- Solving by substitution:
  - Solve one equation for one variable (doesn’t matter which one)
  - Substitute that variable into the other equation
  - Solve new linear equation (should have only one variable)
  - Substitute answer back into earlier equation

Substitution

Problem 2. Solve the system of equations by substitution.

\[
\begin{align*}
  x - 3y &= 12 \\
  -2x - 2y &= 8
\end{align*}
\]

A. (0, -4)
B. (1, -5)
C. (4, 0)
D. No solution
E. Infinitely many solutions
F. None of the above
2.4 Elimination by Addition

Equivalent Systems

- We can get a new equivalent system by:
  - Interchanging two equations
  - Multiplying one equation by a number other than zero
  - Adding a constant multiple of one equation to another

Example.

\[
\begin{align*}
3x + 2y &= 7 \\
5x + y &= -1
\end{align*}
\quad \text{and} \quad
\begin{align*}
14x + 7y &= 20 \\
6x + 4y &= 14
\end{align*}
\]

are equivalent systems.

Elimination by Addition

- To solve using elimination by addition:
  - Choose one variable
  - Multiply both equations by an appropriate nonzero constant so that the coefficients of the variables are opposite
  - Add equations together to get rid of the one variable
  - Solve for the remaining variable
  - Substitute back into either original equation and solve for other variable

Elimination by Addition

Problem 3. Solve the system of equations by elimination.

\[
\begin{align*}
2x + y &= 25 \\
6x - y &= 7
\end{align*}
\]

A. \((-4, 17)\)

B. \((4, 17)\)
3 Applications

Problem 4. Sam and Dave are ticket-sellers for their class play. Sam is selling student tickets for $2.00 each, and Dave is selling adult tickets for $5.50 each. If their total income for 24 tickets is $83.00, how many tickets did Sam sell?

A. 10 tickets
B. 14 tickets
C. 15 tickets
D. 16 tickets
E. None of the above

Summary

You should be able to:

- Solve systems of two linear equations by
  - Graphing
  - Substitution
  - Elimination by addition
- Solve applications involving systems of linear equations