# 3. Assessing Flow Conditions: Subcritical & Supercritical Flow

The Reynolds number, which we explored in assignment 2, allows us to classify stream flow as either laminar or turbulent. The key difference between these flow conditions is whether water molecules are flowing in parallel paths in a downstream direction at a constant velocity or whether water molecules are traveling along paths going in different directions and/or at different velocities. The classification of flow as laminar or turbulent is useful because it allows us to assess the potential for sediment entrainment and erosion along the channel boundary.

The Froude number provides a second way of classifying flow conditions and is based not on the paths that strands of water molecules follow, but on the relationship between flow velocity and flow depth. Similar to the Reynolds number, the Froude number helps assess the energy state of water flow. Another similarity to the Reynolds number is the fact that the Froude number is dimensionless – there are no units associated with the Froude number.

The Froude number is defined as the ratio of gravitational forces to inertial forces:

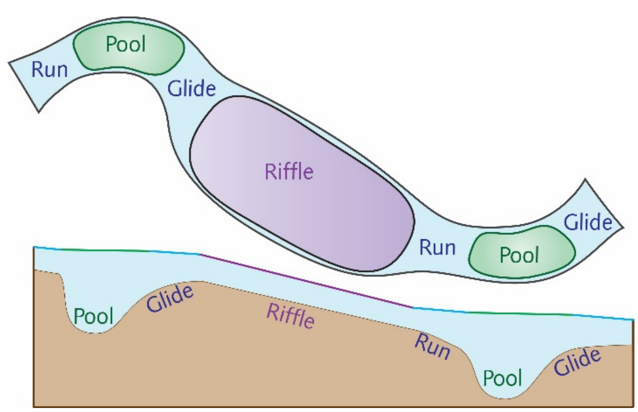
$$F = \frac{gravitational\,forces}{inertial\,forces} = \frac{\bar{v}}{\sqrt{(g \times \bar{d})}}$$

Average flow velocity provides a measure of gravitational forces, while the combination of gravity and depth provides a measure of inertial forces. Inertia is a measure of an object's resistance to a change in motion. Heavy objects or substances have more inertia than light objects or substances; it is much harder to set a heavy object in motion than a light one, and once in motion, a heavy object is much more difficult to redirect, slow down or speed up compared to a light one. The combination of gravity and depth provides a measure of the weight of the water – its inertia. Deep water has considerably more inertia than shallow water. Assuming a constant discharge along a channel, the Froude number tells us that the discharge can be transmitted along the channel as deep slow flow or as shallow fast flow. With deep slow flow, inertial forces dominate flow conditions while with shallow fast flow, gravitational forces dominate flow conditions. .

**Subcritical** flow is deep, slow flow with a low energy state and has a Froude number less than one (F<1). **Critical** flow occurs when the Froude number equals one (F=1); there is a perfect balance between the gravitational and inertial forces. **Supercritical** flow is shallow, fast flow with a high energy state and has a Froude number greater than one (F>1). The energy state of the river affects the potential for entrainment and erosion of the channel boundary as well as affecting the extent to which downstream disturbances to forward flow from boulders or built structures can be transmitted upstream. When flow conditions are subcritical, disturbances in the flow can generate surface waves, and because the downstream flow is so slow, those surface waves can travel in an upstream direction. When flow conditions are supercritical, surface waves generated by downstream disturbances cannot travel upstream; the forward flow of water carries the waves downstream.

Features found in many streams include pools, riffles, glides and runs (Figure 1). These features are defined in part based on depth and flow velocity, and as a result, scientists interested in river restoration and aquatic habitat delineation have used the Froude number to define these features (e.g. Jowett 1993). **Pools** are characterized by deep water, slow flow, a surface slope that is near zero and low Froude numbers. Hilldale (2007) defined pools as areas with Froude numbers less than 0.09, while Jowett (1993) defined pools as areas with Froude numbers less than 0.18. **Riffles** are characterized by shallow water, fast flow, a steep surface slope and Froude numbers greater than 0.41 (Hilldale 2007; Jowett 1993).

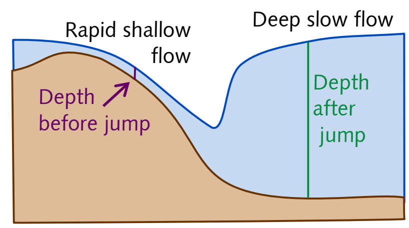
**Figure 1.** Pools, Riffles, Runs and Glides




Note that although the flow conditions in pools and riffles are not the same, they both experience subcritical flow conditions. Supercritical flow, with Froude numbers greater than one, entail *very* fast, *very* shallow water and generally occur only over very short stretches of a river. Rivers with irregular channel boundaries, particularly boundaries that contain large boulders and rock outcrops (e.g. whitewater rapids), may contain a combination of subcritical and supercritical flow conditions. For example, water streaming over a boulder on the channel bed may be supercritical. Downstream of the boulder, flow will revert back to subcritical flow conditions. The transition from supercritical flow to subcritical flow can produce a **hydraulic jump,** a region of extreme turbulence where large amounts of energy are expended and where the potential for sediment entrainment and channel erosion is quite high. In a hydraulic jump, flow velocity drops suddenly and water depth increases suddenly (Figure 2). This sudden change in depth and velocity can produce a **standing wave** – the forward momentum of the water is slowed so quickly that the water expands upward. Hydraulic jumps and standing waves can form at the base of dam spillways, waterfalls, and where reservoir sluice gates release water downstream.

Runs and glides connect pools and riffles. A **run** connects a riffle to a downstream pool and is the area where the river bed drops down into the pool. Flow accelerates as water plunges into the pool, which helps maintain the pool through scouring. Flow in runs is too fast to call the run part of a pool and the depth is too great to call the run part of a riffle. According to Jowett (1993) runs have Froude numbers between 0.18 and 0.41. A **glide** connects a pool exit to the next downstream riffle. The bed slope in a glide is negative although the water surface slope is positive. Flow is too shallow to call the glide part of a pool and velocity is too slow to call the glide part of a riffle.

**Figure 2.** A Hydraulic Jump




The objective of this assignment is gain an understanding of how channel geometry and stream flow conditions impact the Froude number.

## References

Hilldale, R.C. 2007. Identifying stream habitat features with a two-dimensional hydraulic model. *Technical Series TS-YSS-12*, Bureau of Reclamation Technical Service Center, Denver, CO. 33 pp.

Jowett, I.G. 1993. A method for identifying pool, run, and riffle habitats from physical measurements. *New Zealand Journal of Marine and Freshwater Research* 27:242-248.

## Learning Outcomes

By the end of this assignment you should be able to:

- Calculate the Froude number;
- Explain how and why different combinations of stream channel geometry, flow velocity, and discharge affect the flow conditions represented by the Froude number;
- Assess the nature of stream flow conditions represented by different Froude numbers.

## Symbols

$\overline{v}$ = average velocity (ft/s)

$\overline{d}$ = average channel depth

g = acceleration of gravity = 32.2 ft/s/s

g = 9.81 m/s/s

## Equations

$Q$ = discharge = $w \times \overline{d} \times \overline{v} = A \times \overline{v}$

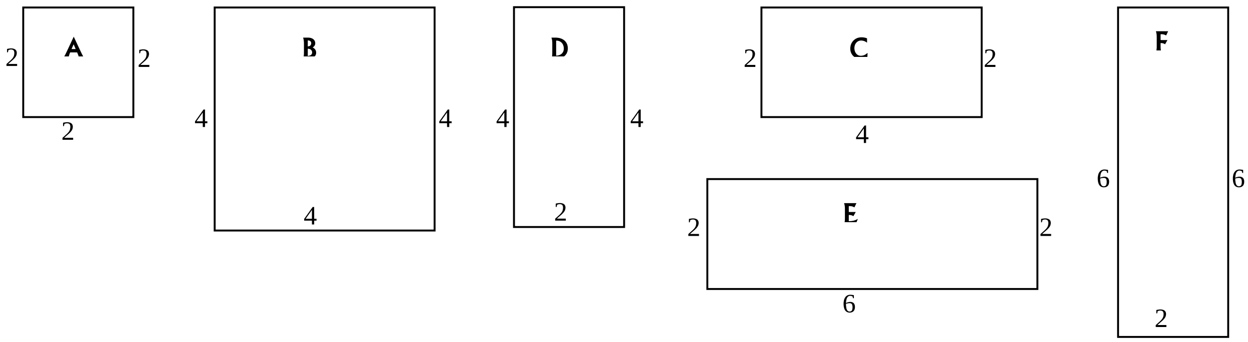
$F$ = Froude number = $\overline{v} \,/\, \sqrt{(g \times \overline{d})}$

Name ______________________________ 42 Points

# Assessment of Flow Conditions: Subcritical & Supercritical Flow

## Information

| | A | B | C | D | E | F |
|---|---|---|---|---|---|---|
| Average depth, $\bar{d}$ | 2 | 4 | 2 | 4 | 2 | 6 |
| Channel width, w | 2 | 4 | 4 | 2 | 6 | 2 |
| Cross-sectional area, a | 4 | 16 | 8 | 8 | 12 | 12 |
| Wetted perimeter, $P_w$ | 6 | 12 | 8 | 10 | 10 | 14 |
| Hydraulic radius, r | 0.7 | 1.3 | 1 | 0.8 | 1.2 | 0.9 |




1.. Calculate the Froude number for each channel assuming $\bar{v}$, is 9.6 ft/s. [6]

Channel A ____________________ Channel D ____________________

Channel B ____________________ Channel E ____________________

Channel C ____________________ Channel F ____________________

2. Based on your calculations in Question 1, rank the six channels, in order, from the channel in which will it be easiest for ripples to travel upstream to the channel in which it will be hardest for ripples to travel upstream. [2]

   Easiest: ⟶ Hardest

3. Channels A, C and E have the ***same depth and velocity*** but ***different cross-sectional areas.*** Based on your calculations in Question 1, how does the difference in cross-sectional area affect the Froude number and thus the ease with which ripples can travel upstream? Why? [2]

4. Channels C and D have the ***same cross-sectional area and velocity*** but ***different depths.*** Based on your calculations in Question 1, how does the difference in depth affect the Froude number and thus the ease with which ripples can travel upstream? Why? [2]

5. Calculate the average flow velocity and the Froude number for channels A, B, C, and D assuming discharge (Q) equals 50 $ft^3/s$. [8]

| | Velocity | Froude # |
|---|---|---|
| A | ____________ | ____________ |
| B | ____________ | ____________ |
| C | ____________ | ____________ |
| D | ____________ | ____________ |

6. a. Channels B and D have the ***same depth and discharge*** but ***different cross-sectional areas and different Froude numbers***. Based on your calculations in Question 5, what does the difference in the Froude number tell you about the difference in flow conditions for these two channels? [2]

   b. If cross-sectional area doesn't affect the Froude number, why are the Froude numbers, and thus the flow conditions, for channels B and D different? [2]

7. a. Channels C and D have the same discharge, cross-sectional area, and flow velocity, but their Froude numbers are different. Why? [1]

   b. What does the difference in the Froude number tell you about the difference in flow conditions between channels C and D? [1]

8. In general, for a given discharge, where should it be more difficult for ripples to travel upstream, in wide shallow channels or in narrow deep channels? Why? [2]

9. Based on your calculations, how does flow velocity affect the Froude number and thus flow conditions? [1]

10. Based on your calculations, how does channel depth affect the Froude number and thus flow conditions? [1]

11. Based on your calculations, how does cross-sectional area affect the Froude number and thus flow conditions? [1]

12. Based on your calculations, how does channel width affect the Froude number and thus flow conditions? [1]

Note that units in the next two questions are metric!

13. If you want to create a pool habitat in a stream, defined by $F< 0.18$, what is the minimum depth needed if the average stream velocity is: [2]

    0.74 m/s? _______________

    0.5 m/s? _______________

14. As part of a stream restoration project, you want to create some pool and riffle habitats in a stream with an average discharge of 1.6 $m^3/s$. The maximum width of the restored stream cannot exceed 6 m. Calculate the depth and the Froude number for the following velocities and identify whether the conditions define a pool, a riffle, or a glide. [8]

    Constants: $Q=1.6\ m^3/s$ $w=6\ m$

    a. $\overline{v}$ = 0.2 m/s d = __________ F = __________ Habitat = _____________

    b. $\overline{v}$ = 0.4 m/s d = __________ F = __________ Habitat = _____________

    c. $\overline{v}$ = 0.6 m/s d = __________ F = __________ Habitat = _____________

    d. $\overline{v}$ = 0.8 m/s d = __________ F = __________ Habitat = _____________

    e. $\overline{v}$ = 1.0 m/s d = __________ F = __________ Habitat = _____________