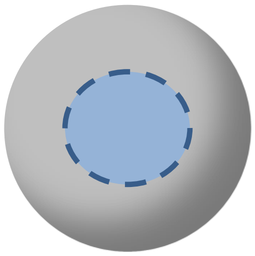
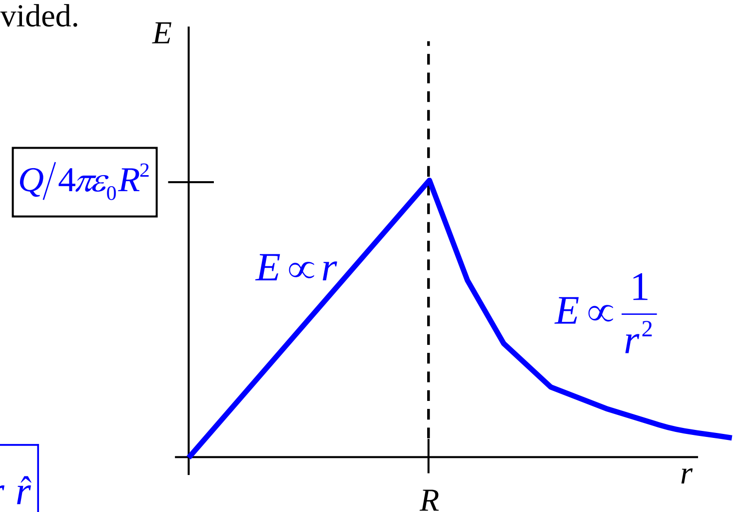
Gauss's Law Geometries
Physics 320 Prof. Menningen

Spherical Geometry (Griffiths 2.12, POP4 19.40.mod) An insulating solid sphere of radius R has a uniform charge density ρ_e and carries a total positive charge Q. Plot the electric field as a function of r. Write E_{max} in symbolic form in the box provided.





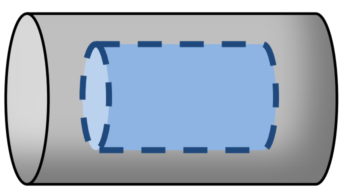
Inside: $\oint \vec{E}\cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0} = \frac{1}{\varepsilon_0} Q \frac{\frac{4}{3}\pi r^3}{\frac{4}{3}\pi R^3}$

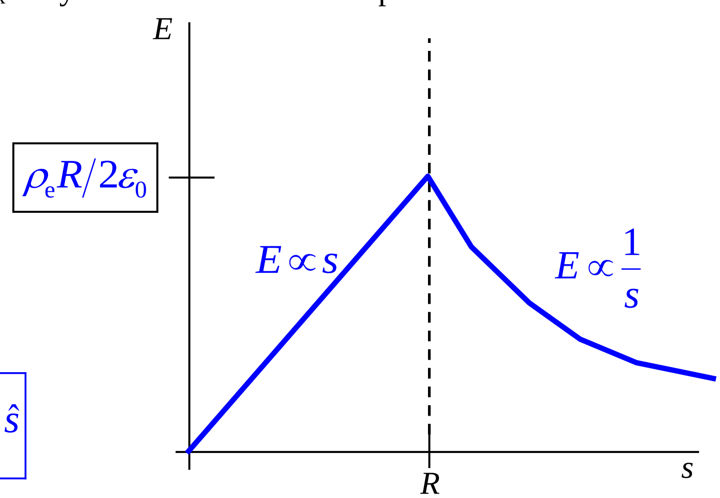
$E\left(4\pi r^2\right) = \frac{Q}{\varepsilon_0}\frac{r^3}{R^3} \quad \Rightarrow \vec{E} = \boxed{\frac{Q}{4\pi\varepsilon_0 R^3} r\ \hat{r}}$

Outside: $\oint \vec{E}\cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0} = \frac{Q}{\varepsilon_0}$

$E\left(4\pi r^2\right) = \frac{Q}{\varepsilon_0} \quad \Rightarrow \vec{E} = \boxed{\frac{Q}{4\pi\varepsilon_0 r^2}}\hat{r}$

Cylindrical Geometry (Griffiths Example 2.3mod, POP4 19.39) Consider a long cylindrical charge distribution of radius R with a uniform charge density ρ_e. Find the magnitude of the electric field everywhere and graph it below. Write E_{max} in symbolic form in the box provided.





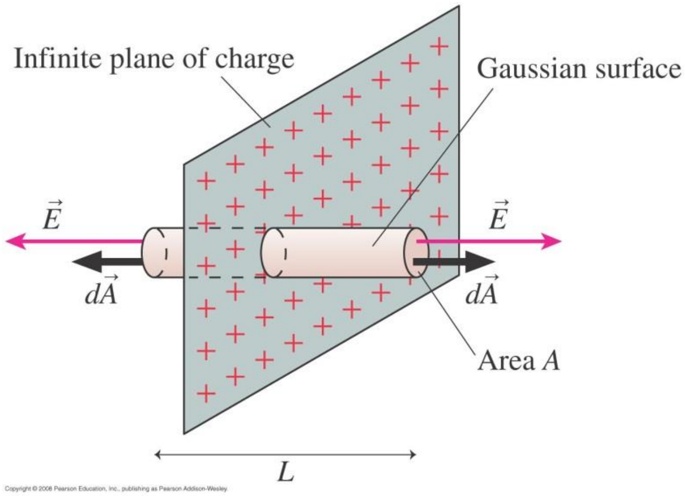
Inside: $\oint \vec{E}\cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$

$E(2\pi sL) = \frac{\rho\left(\pi s^2 L\right)}{\varepsilon_0} \quad \Rightarrow \vec{E} = \boxed{\frac{\rho}{2\varepsilon_0} s\ \hat{s}}$

Outside: $\oint \vec{E}\cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$

$E(2\pi sL) = \frac{\rho\left(\pi R^2 L\right)}{\varepsilon_0} \quad \Rightarrow \vec{E} = \boxed{\frac{\rho R^2}{2\varepsilon_0}\frac{1}{s}\ \hat{s}}$

Planar Geometry (Griffiths 2.17, like PSE6 24.37) An infinite plane slab, of thickness $2d$, carries a uniform volume charge density ρ (Fig. 2.27). Find the electric field, as a function of y, where $y = 0$ at the center. Plot E versus y, calling E positive when it points in the $+y$ direction and negative when it points in the $-y$ direction.



$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{enc}}}{\varepsilon_0}$$

inside:

$$EA + 0 + EA = \frac{\rho(2y)A}{\varepsilon_0}$$

$$\vec{E} = \frac{\rho y}{\varepsilon_0}\begin{cases} \hat{y} & \text{for } y > 0 \\ -\hat{y} & \text{for } y < 0 \end{cases}$$

outside:

$$EA + 0 + EA = \frac{\rho(2d)A}{\varepsilon_0}$$

$$\vec{E} = \frac{\rho d}{\varepsilon_0}\begin{cases} \hat{y} & \text{for } y > 0 \\ -\hat{y} & \text{for } y < 0 \end{cases}$$

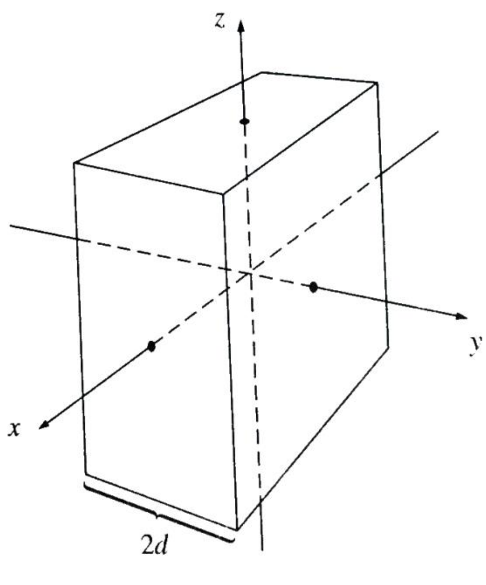


Figure 2.27