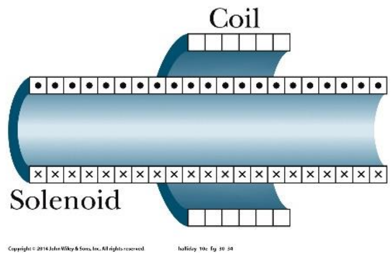
Homework Chapter 30: Induction and Inductance

30.03 In Fig. 30-34, a 120-turn coil of radius 1.8 cm and resistance 5.3 Ω is coaxial with a solenoid of 220 turns/cm and diameter 3.2 cm. The solenoid current drops from 1.5 A to zero in time interval Δt = 25 ms. What current is induced in the coil during Δt?



3. **THINK** Changing the current in the solenoid changes the flux, and therefore, induces a current in the coil.

EXPRESS Using Faraday's law, the total induced emf is given by

$$\varepsilon = -N\frac{d\Phi_B}{dt} = -NA\left(\frac{dB}{dt}\right) = -NA\frac{d}{dt}(\mu_0 ni) = -N\mu_0 nA\frac{di}{dt} = -N\mu_0 n(\pi r^2)\frac{di}{dt}$$

By Ohm's law, the induced current in the coil is $i_{\text{ind}} = |\varepsilon|/R$, where R is the resistance of the coil.

Use the solenoid radius because the field outside of it is zero and contributes no change in flux through the coil.

ANALYZE Substituting the values given, we obtain

$$\varepsilon = -N\mu_0 n(\pi r^2)\frac{di}{dt} = -(120)(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A})(22000/\text{m})\,\pi(0.016\,\text{m})^2\left(\frac{1.5\text{ A}}{0.025\text{ s}}\right)$$
$$= 0.16\text{V}.$$

Ohm's law then yields $i_{\text{ind}} = \frac{|\varepsilon|}{R} = \frac{0.016\text{ V}}{5.3\,\Omega} = 0.030\text{ A}.$

LEARN The direction of the induced current can be deduced from Lenz's law, which states that the direction of the induced current is such that the magnetic field which it produces opposes the change in flux that induces the current.

30.09 A small loop of area 6.8 mm^2 is placed inside a long solenoid that has 854 turns/cm and carries a sinusoidally varying current i of amplitude 1.28 A and angular frequency 212 rad/s. The central axes of the loop and solenoid coincide. What is the amplitude of the emf induced in the loop?

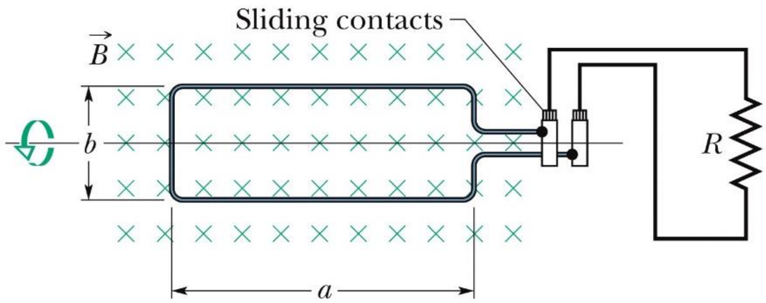
9. The amplitude of the induced emf in the loop is

$$\varepsilon_m = A\mu_0 n i_0 \omega = (6.8\times10^{-6}\text{m}^2)(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A})(85400/\text{m})(1.28\text{ A})(212\text{ rad/s})$$
$$= 1.98\times10^{-4}\text{V}.$$

30.11 A rectangular coil of N turns and of length a and width b is rotated at frequency f in a uniform magnetic field $\vec{\mathbf{B}}$, as indicated in Fig. 30-40. The coil is connected to co-rotating cylinders, against which metal brushes slide to make contact. (a) Show that the emf induced in the coil is given (as a function of time t) by

$$\mathcal{E} = 2\pi fNabB \sin(2\pi ft) = \mathcal{E}_0 \sin(2\pi ft).$$

This is the principle of the commercial alternating-current generator. (b) What value of Nab gives an emf with $\mathcal{E}_0 = 150$ V when the loop is rotated at 60.0 rev/s in a uniform magnetic field of 0.500 T?



11. (a) It should be emphasized that the result, given in terms of $\sin(2\pi ft)$, could as easily be given in terms of $\cos(2\pi ft)$ or even $\cos(2\pi ft + \phi)$ where ϕ is a phase constant as discussed in Chapter 15. The angular position θ of the rotating coil is measured from some reference line (or plane), and which line one chooses will affect whether the magnetic flux should be written as $BA \cos\theta$, $BA \sin\theta$ or $BA \cos(\theta + \phi)$. Here our choice is such that $\Phi_B = BA\cos\theta$. Since the coil is rotating steadily, θ increases linearly with time. Thus, $\theta = \omega t$ (equivalent to $\theta = 2\pi ft$) if θ is understood to be in radians (and ω would be the angular velocity). Since the area of the rectangular coil is $A=ab$, Faraday's law leads to

$$\varepsilon = -N\frac{d(BA\cos\theta)}{dt} = -NBA\frac{d\cos(2\pi ft)}{dt} = N\,Bab2\pi f \sin(2\pi ft)$$

which is the desired result, shown in the problem statement. The second way this is written ($\varepsilon_0 \sin(2\pi ft)$) is meant to emphasize that the voltage output is sinusoidal (in its time dependence) and has an amplitude of $\varepsilon_0 = 2\pi f NabB$.

(b) We solve

$$\varepsilon_0 = 150 \text{ V} = 2\pi f NabB$$

when $f = 60.0$ rev/s and $B = 0.500$ T. The three unknowns are N, a, and b which occur in a product; thus, we obtain $Nab = 0.796 \text{ m}^2$.

30.17 A small circular loop of area 2.00 cm^2 is placed in the plane of, and concentric with, a large circular loop of radius 1.00 m. The current in the large loop is changed at a constant rate from 200 A to −200 A (a change in direction) in a time of 1.00 s, starting at $t = 0$. What is the magnitude of the magnetic field $\vec{\mathbf{B}}$ at the center of the small loop due to the current in the large loop at (a) $t = 0$, (b) $t = 0.500$ s, and (c) $t = 1.00$ s? (d) From $t = 0$ to $t = 1.00$ s, is $\vec{\mathbf{B}}$ reversed? Because the inner loop is small, assume $\vec{\mathbf{B}}$ is uniform over its area. (e) What emf is induced in the small loop at $t = 0.500$ s?

17. Equation 29-10 gives the field at the center of the large loop with R = 1.00 m and current $i(t)$. This is approximately the field throughout the area (A = 2.00 × 10^{-4} m^2) enclosed by the small loop. Thus, with $B = \mu_0 i/2R$ and $i(t) = i_0 + kt$, where i_0 = 200 A and

$$k = (–200 \text{ A} – 200 \text{ A})/1.00 \text{ s} = – 400 \text{ A/s},$$

we find

(a) $B(t=0)=\dfrac{\mu_0 i_0}{2R}=\dfrac{(4\pi\times10^{-7}\,\text{H/m})(200\text{A})}{2(1.00\text{m})}=1.26\times10^{-4}\,\text{T},$

(b) $B(t=0.500\text{s})=\dfrac{(4\pi\times10^{-7}\,\text{H/m})[200\text{A}-(400\text{A/s})(0.500\text{s})]}{2(1.00\text{m})}=0,$ and

(c) $B(t=1.00\text{s})=\dfrac{(4\pi\times10^{-7}\,\text{H/m})[200\text{A}-(400\text{A/s})(1.00\text{s})]}{2(1.00\text{m})}=-1.26\times10^{-4}\,\text{T},$

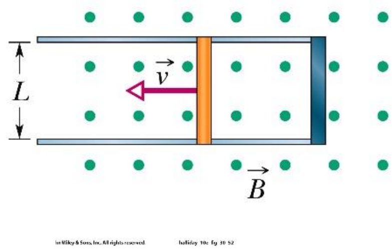
or $|B(t=1.00\text{s})|=1.26\times10^{-4}\,\text{T}.$

(d) Yes, as indicated by the flip of sign of $B(t)$ in (c).

(e) Let the area of the small loop be a. Then $\Phi_B = Ba$, and Faraday's law yields

$$\begin{aligned}\varepsilon &= -\frac{d\Phi_B}{dt} = -\frac{d(Ba)}{dt} = -a\frac{dB}{dt} = -a\left(\frac{\Delta B}{\Delta t}\right)\\ &= -(2.00\times10^{-4}\text{m}^2)\left(\frac{-1.26\times10^{-4}\,\text{T}-1.26\times10^{-4}\,\text{T}}{1.00\text{ s}}\right)\\ &= 5.04\times10^{-8}\text{ V}.\end{aligned}$$

30.29 In Fig. 30-52, a metal rod is forced to move with constant velocity $\vec{\mathbf{v}}$ along two parallel metal rails, connected with a strip of metal at one end. A magnetic field of magnitude B = 0.350 T points out of the page. (a) If the rails are separated by L = 25.0 cm and the speed of the rod is 55.0 cm/s, what emf is generated? (b) If the rod has a resistance of 18.0 Ω and the rails and connector have negligible resistance, what is the current in the rod? (c) At what rate is energy being transferred to thermal energy?



29. (a) Equation 30-8 leads to

$$\varepsilon = BLv = (0.350 \text{ T})(0.250 \text{ m})(0.55 \text{ m/s}) = 0.0481 \text{ V}.$$

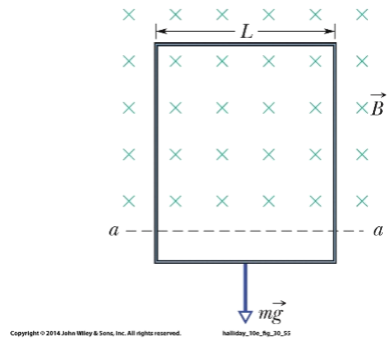
(b) By Ohm's law, the induced current is

$$i = 0.0481 \text{ V}/18.0\ \Omega = 0.00267 \text{ A}.$$

By Lenz's law, the current is clockwise in Fig. 30-52.

(c) Equation 26-27 leads to $P = i^2R$ = 0.000129 W.

30.34 In Fig. 30-55, a long rectangular conducting loop, of width L, resistance R, and mass m, is hung in a horizontal, uniform magnetic field $\vec{\mathbf{B}}$ that is directed into the page and that exists only above line aa. The loop is then dropped; during its fall, it accelerates until it reaches a certain terminal speed v_t. Ignoring air drag, find an expression for v_t.



34. Noting that $F_{net} = BiL - mg = 0$, we solve for the current:

$$i = \frac{mg}{BL} = \frac{|\varepsilon|}{R} = \frac{1}{R}\left|\frac{d\Phi_B}{dt}\right| = \frac{B}{R}\left|\frac{dA}{dt}\right| = \frac{Bv_t L}{R},$$

which yields $v_t = mgR/B^2L^2$.

Skipped steps: begin by setting the forces equal (implied above by $F_{net} = 0$)

$$F_{magnetic} = F_{gravitational}$$
$$BiL = mg$$
$$B\left(\frac{Bv_t L}{R}\right)L = mg$$
$$v_t = \frac{mgR}{B^2L^2}$$

Sometimes it's helpful to do unit analysis.

Because $B = \frac{F_{magnetic}}{qv}$ we know 1 Tesla $= \frac{\text{N}}{\text{C}\cdot\text{m/s}} = \frac{\text{N}\cdot\text{s}}{\text{C}\cdot\text{m}}$.

Also, $R = \frac{V}{I} = \frac{\text{J/C}}{\text{C/s}} = \frac{\text{J}\cdot\text{s}}{\text{C}^2} = \frac{\text{N}\cdot\text{m}\cdot\text{s}}{\text{C}^2}$. Therefore:

$\frac{mgR}{B^2L^2} = \text{N}\frac{\text{N}\cdot\text{m}\cdot\text{s}}{\text{C}^2}\frac{\text{C}^2\cdot\text{m}^2}{\text{N}^2\cdot\text{s}^2}\frac{1}{\text{m}^2} = \frac{\text{m}}{\text{s}}$, the expected units for v_t

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30.40 The inductance of a closely packed coil of 400 turns is 8.0 mH. Calculate the magnetic flux through the coil when the current is 5.0 mA.

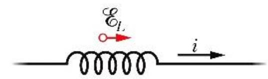
What is shown below is the flux through *each turn* of the coil.

40. Since $N\Phi_B = Li$, we obtain

$$\Phi_B = \frac{Li}{N} = \frac{(8.0\times10^{-3}\ \text{H})(5.0\times10^{-3}\ \text{A})}{400} = 1.0\times10^{-7}\ \text{Wb}.$$

The total flux through the coil would be this value multiplied by N:
$N\Phi_B = (400)(1.0\times10^{-7}\ \text{Wb}) =$
$= 4.0\times10^{-5}\ \text{Wb} = 40\ \mu\text{Wb}$

30.45 At a given instant the current and self-induced emf in an inductor are directed as indicated in Fig. 30-59. (a) Is the current increasing or decreasing? (b) The induced emf is 17 V, and the rate of change of the current is 25 kA/s; find the inductance.



45. (a) Speaking anthropomorphically, the coil wants to fight the changes—so if it wants to push current rightward (when the current is already going rightward) then i must be in the process of decreasing.

(b) From Eq. 30-35 (in absolute value) we get

$$L = \left|\frac{\varepsilon}{di/dt}\right| = \frac{17\,\text{V}}{2.5\,\text{kA/s}} = 6.8\times10^{-4}\,\text{H}.$$

supposed to be 25 kA/s. The displayed answer is correct.

30.53 A solenoid having an inductance of 6.30 μH is connected in series with a 1.20 kΩ resistor. (a) If a 14.0 V battery is connected across the pair, how long will it take for the current through the resistor to reach 80.0% of its final value? (b) What is the current through the resistor at time $t = 1.0\tau_L$?

53. **THINK** The inductor in the *RL* circuit initially acts to oppose changes in current through it.

EXPRESS If the battery is switched into the circuit at $t = 0$, then the current at a later time t is given by

$$i = \frac{\varepsilon}{R}\left(1 - e^{-t/\tau_L}\right),$$

where $\tau_L = L/R$.

(a) We want to find the time at which $i = 0.800\varepsilon/R$. This means

$$0.800 = 1 - e^{-t/\tau_L} \;\Rightarrow\; e^{-t/\tau_L} = 0.200.$$

Taking the natural logarithm of both sides, we obtain

$$-(t/\tau_L) = \ln(0.200) = -1.609.$$

Thus,

$$t = 1.609\tau_L = \frac{1.609L}{R} = \frac{1.609(6.30\times10^{-6}\,\text{H})}{1.20\times10^{3}\,\Omega} = 8.45\times10^{-9}\,\text{s}.$$

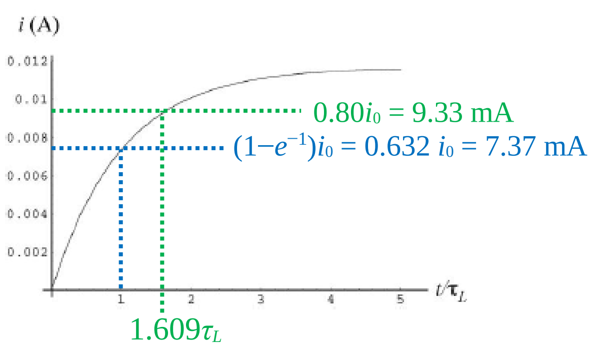
(b) At $t = 1.0\,\tau_L$ the current in the circuit is

$$i = \frac{\varepsilon}{R}\left(1 - e^{-1.0}\right) = \left(\frac{14.0\,\text{V}}{1.20\times10^{3}\,\Omega}\right)\left(1 - e^{-1.0}\right) = 7.37\times10^{-3}\ \text{A}.$$

LEARN At $t = 0$, the current in the circuit is zero. However, after a very long time, the inductor acts like an ordinary connecting wire, so the current is

$$i_0 = \frac{\varepsilon}{R} = \frac{14.0\,\text{V}}{1.20\times10^{3}\,\Omega} = 0.0117\ \text{A}.$$

The current as a function of t/τ_L is plotted to the right.



30.64 At $t = 0$, a battery is connected to a series arrangement of a resistor and an inductor. At what multiple of the inductive time constant will the energy stored in the inductor's magnetic field be 0.500 its steady-state value?

64. Let $U_B(t)=\frac{1}{2}Li^2(t)$. We require the energy at time t to be half of its final value: $U(t)=\frac{1}{2}U_B(t\to\infty)=\frac{1}{4}Li_f^2$. This gives $i(t)=i_f/\sqrt{2}$. But $i(t)=i_f(1-e^{-t/\tau_L})$, so

$$1-e^{-t/\tau_L}=\frac{1}{\sqrt{2}} \quad\Rightarrow\quad \frac{t}{\tau_L}=-\ln\left(1-\frac{1}{\sqrt{2}}\right)=1.23.$$

30.67 A solenoid that is 85.0 cm long has a cross-sectional area of 17.0 cm^2. There are 950 turns of wire carrying a current of 6.60 A. (a) Calculate the energy density of the magnetic field inside the solenoid. (b) Find the total energy stored in the magnetic field there (neglect end effects).

67. **THINK** The magnetic energy density is given by $u_B = B^2/2\mu_0$, where B is the magnitude of the magnetic field at that point.

EXPRESS Inside a solenoid, the magnitude of the magnetic field is $B = \mu_0 n i$, where

$$n = (950 \text{ turns})/(0.850 \text{ m}) = 1.118 \times 10^3 \text{ m}^{-1}.$$

Thus, the energy density is

$$u_B=\frac{B^2}{2\mu_0}=\frac{(\mu_0 n i)^2}{2\mu_0}=\frac{1}{2}\mu_0 n^2 i^2.$$

Since the magnetic field is uniform inside an ideal solenoid, the total energy stored in the field is $U_B = u_B\mathcal{V}$, where $\mathcal{V}$ is the volume of the solenoid.

ANALYZE (a) Substituting the values given, we find the magnetic energy density to be

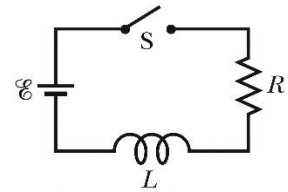
$$u_B=\frac{1}{2}\mu_0 n^2 i^2=\frac{1}{2}(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A})(1.118\times10^3\,\text{m}^{-1})^2(6.60\,\text{A})^2=34.2\,\text{J/m}^3 .$$

(b) The volume $\mathcal{V}$ is calculated as the product of the cross-sectional area and the length. Thus,

$$U_B=(34.2\,\text{J/m}^3)(17.0\times10^{-4}\,\text{m}^2)(0.850\,\text{m})=4.94\times10^{-2}\,\text{J} .$$

LEARN Note the similarity between $u_B=\frac{B^2}{2\mu_0}$, the energy density at a point in a magnetic field, and $u_E=\frac{1}{2}\varepsilon_0 E^2$, the energy density at a point in an electric field. Both quantities are proportional to the square of the fields.

30.83 Switch S in Fig. 30-63 is closed at time $t = 0$, initiating the buildup of current in the 15.0 mH inductor and the 20.0 Ω resistor. At what time is the emf across the inductor equal to the potential difference across the resistor?



83. Equation 30-41 applies, and the problem requires

$$iR = L\frac{di}{dt} = \varepsilon - iR$$

at some time t (where Eq. 30-39 has been used in that last step). Thus, we have $2iR = \varepsilon$, or

$$\varepsilon = 2iR = 2\left[\frac{\varepsilon}{R}(1-e^{-t/\tau_L})\right]R = 2\varepsilon\left(1-e^{-t/\tau_L}\right)$$

where Eq. 30-42 gives the inductive time constant as $\tau_L = L/R$. We note that the emf ε cancels out of that final equation, and we are able to rearrange (and take the natural log) and solve. We obtain $t = 0.520$ ms.

The solution manual sets the voltage iR across the resistor equal to half the emf $\mathcal{E}$ and works from there. Alternatively, we could set the inductor and resistor voltages equal to each other and work from there:

$$\Delta V_L = \Delta V_R$$

$$L\frac{di}{dt} = iR$$

$$L\frac{d}{dt}\left[\frac{\mathcal{E}}{R}\left(1-e^{-t/\tau_L}\right)\right] = \left[\frac{\mathcal{E}}{R}\left(1-e^{-t/\tau_L}\right)\right]R$$

$$\frac{L\mathcal{E}}{R}\left[+\frac{1}{\tau_L}e^{-t/\tau_L}\right] = \mathcal{E}\left(1-e^{-t/\tau_L}\right) \quad \text{but } \tau_L = \frac{L}{R} \quad \text{so}$$

$$\left[e^{-t/\tau_L}\right] = \left(1-e^{-t/\tau_L}\right)$$

$$2e^{-t/\tau_L} = 1$$

$$-\frac{t}{\tau_L} = \ln\left(\frac{1}{2}\right)$$

$$t = \tau_L \ln(2) = \left(\frac{0.015\text{ H}}{20.0\ \Omega}\right)\ln(2) = (0.750\text{ ms})\ln(2) = \boxed{0.520\text{ ms}}$$