# Discussion Examples
## Chapter 13: Oscillations About Equilibrium

17. The position of a mass on a spring is given by $x = (6.5\text{ cm})\ \cos\left[2\pi t/(0.88\text{ s})\right]$. **(a)** What is the period, *T*, of this motion? **(b)** Where is the mass at $t = 0.25\text{ s}$? **(c)** Show that the mass is at the same location at $0.25\text{ s} + T$ seconds as it is at 0.25 s.

**Picture the Problem**: A mass is attached to a spring. The mass is displaced from equilibrium and released from rest. The spring force causes the mass to oscillate about the equilibrium position in harmonic motion.

**Strategy:** The period can be obtained directly from the argument of the cosine function. Substituting the specific time into the equation will yield the location at that time. Substituting in the new time will show that the mass is at the same location one period later.

**Solution: 1**. **(a)** Identify the period (*T*) from the cosine equation:

$$x = A\cos\left[\left(\frac{2\pi}{T}\right)t\right],\ \text{so here } T = \boxed{0.88\text{ s}}.$$

**2. (b)** Substitute *t* = 0.25 s into the equation and evaluate *x*:

$$x = (6.5\text{ cm})\cos\left[\left(\frac{2\pi}{0.88\text{ s}}\right)0.25\text{ s}\right] = \boxed{-1.4\text{ cm}}$$

**3. (c)** Substitute *t* = (0.25 s + *T*) into the equation and factor:

$$x = A\cos\left[\left(\frac{2\pi}{T}\right)(0.25\text{ s} + T)\right] = A\cos\left[\left(\frac{2\pi}{T}\right)(0.25\text{ s} + 2\pi)\right]$$

**4.** Drop the $2\pi$ phase shift, because $\cos(x + 2\pi) = \cos(x)$:

$$x = A\cos\left[\left(\frac{2\pi}{T}\right)0.25\text{ s}\right]$$

**5.** Insert the numeric values:

$$x = (6.5\text{ cm})\cos\left[\left(\frac{2\pi}{0.88\text{ s}}\right)0.25\text{ s}\right] = \boxed{-1.4\text{ cm}} \Rightarrow \boxed{\text{same location}}$$

**Insight:** Increasing the time by any multiple of the period increases the argument of the cosine function by the same multiple of $2\pi$, which has no effect upon the value of the cosine function.

44. **IP** The springs of a 511-kg motorcycle have an effective force constant of 9130 N/m. **(a)** If a person sits on the motorcycle, does its period of oscillation increase, decrease, or stay the same? **(b)** By what percent and in what direction does the period of oscillation change when a 112-kg person rides the motorcycle?

**Picture the Problem**: If the motorcycle is pushed down slightly on its springs it will oscillate up and down in harmonic motion. A rider sitting on the motorcycle effectively increases the mass of the motorcycle and oscillates also.

**Strategy:** We can use the equation for the period of a mass on a spring. Writing this equation for the motorcycle without rider and again for the motorcycle with rider we can calculate the percent difference in the periods.

**Solution: 1. (a)** The period $\boxed{\text{increases,}}$ because the person's mass is added to the system and $T \propto \sqrt{m}$.

**2. (b)** Write the equation for the period of the motorcycle without the rider:

$$T = 2\pi\sqrt{\frac{m}{k}}$$

**3.** Write the equation for the period of the motorcycle with the rider:

$$T_2 = 2\pi\sqrt{\frac{m+M}{k}}$$

**4.** Calculate the percent difference between the two periods:

$$\frac{T_2 - T}{T} = \frac{2\pi\sqrt{\frac{m+M}{k}} - 2\pi\sqrt{\frac{m}{k}}}{2\pi\sqrt{\frac{m}{k}}}$$

**5.** Simplify by factoring out $2\pi\sqrt{m/k}$ from the numerator and denominator:

$$\frac{T_2 - T}{T} = \sqrt{\frac{m+M}{m}} - 1$$

$$= \sqrt{\frac{511 + 122\text{ kg}}{511\text{ kg}}} - 1 = 0.104 = \boxed{10.4\%}$$

**Insight:** The percent change in the period does not depend on the spring force constant. It only depends on the fractional increase in mass.

50. **IP** A 0.40-kg mass is attached to a spring with a force constant of 26 N/m and released from rest a distance of 3.2 cm from the equilibrium position of the spring. **(a)** Give a strategy that allows you to find the speed of the mass when it is halfway to the equilibrium position. **(b)** Use your strategy to find this speed.

**Picture the Problem**: A mass attached to a spring is stretched from equilibrium position.

**Strategy:** The work done in stretching the spring is stored as potential energy in the spring until the mass is released. After the mass is released, the mass will accelerate, converting the potential energy into kinetic energy. The energy will then transfer back and forth between potential and kinetic energies as the mass oscillates about the equilibrium position. Solve the conservation of mechanical energy equation, $E = K + U$, for the kinetic energy. Then use the equation for kinetic energy, $K = \frac{1}{2}mv^2$, to solve for the velocity.

**Solution: 1.** Set $K = E - U$ and substitute expressions for each term:

$$K = E - U$$

$$\tfrac{1}{2}mv^2 = \tfrac{1}{2}kA^2 - \tfrac{1}{2}k\left(\tfrac{1}{2}A\right)^2$$

**2.** Solve for the speed and simplify:

$$v = \sqrt{\frac{k\left[A^2 - \left(\frac{1}{2}A\right)^2\right]}{m}} = \sqrt{\frac{3kA^2}{4m}}$$

**3.** Insert the numeric values:

$$v = \sqrt{\frac{3(26\text{ N/m})(0.032\text{ m})^2}{4(0.40\text{ kg})}} = \boxed{0.22\text{ m/s}}$$

**Insight:** When the displacement is half the maximum displacement, the speed is not half the maximum speed. In fact, the speed is $\frac{\sqrt{3}}{2}v_{\text{max}}$, which is greater than half the speed.

54. **IP** A 0.505-kg block slides on a frictionless horizontal surface with a speed of 1.18 m/s. The block encounters an unstretched spring and compresses it 23.2 cm before coming to rest. **(a)** What is the force constant of this spring? **(b)** For what length of time is the block in contact with the spring before it comes to rest? **(c)** If the force constant of the spring is increased, does the time required to stop the block increase, decrease, or stay the same? Explain.

**Picture the Problem**: A block has kinetic energy as it slides on a frictionless horizontal surface. It encounters an unstretched spring and compresses it before coming to rest.

**Strategy:** When the mass first encounters the spring the energy is all kinetic. As the spring is compressed the energy is converted to spring potential energy. Equate the energies in order to solve for the spring force constant. The motion corresponds to one-fourth of a period. A stiffer spring will cause the mass to stop in a shorter distance, and therefore a shorter time period.

**Solution: 1. (a)** Equate the kinetic and potential energies:

$$K_{\text{i}} = U_{\text{f}} \quad \Rightarrow \quad \tfrac{1}{2}mv_{\text{i}}^2 = \tfrac{1}{2}kA^2$$

**2.** Solve for the spring force constant:

$$k = m\left(\frac{v_{\text{i}}}{A}\right)^2$$

**3.** Insert the numeric values:

$$k = 0.505\text{ kg}\left(\frac{1.18\text{ m/s}}{0.232\text{ m}}\right)^2 = \boxed{13.1\text{ N/m}}$$

**4. (b)** Solve for one-quarter period:

$$\frac{1}{4}T = \frac{\pi}{2}\sqrt{\frac{m}{k}} = \frac{\pi}{2}\sqrt{\frac{0.505\text{ kg}}{13.06\text{ N/m}}} = \boxed{0.309\text{ s}}$$

**5. (c**) When the force constant increases the time to stop $\boxed{\text{decreases}}$. A greater force constant means a stiffer spring and a greater stopping force, therefore a shorter stopping time.

**Insight:** After the spring has stopped the block all of the energy is in the compressed spring. However, the spring will push back on the block, accelerating it back the way it came. When the block leaves the spring it will have the same speed, but travel in the opposite direction as when it first encountered the spring. The accelerating time is equal to the stopping time.

61. **United Nations Pendulum** A large pendulum with a 200-lb gold-plated bob 12 inches in diameter is on display in the lobby of the United Nations building. The pendulum has a length of 75 ft. It is used to show the rotation of the Earth—for this reason it is referred to as a Foucault pendulum. What is the least amount of time it takes for the bob to swing from a position of maximum displacement to the equilibrium position of the pendulum? (Assume that the acceleration due to gravity is $g = 9.81\ \text{m/s}^2$ at the UN building.)

**Picture the Problem**: The pendulum mass is displaced slightly from equilibrium and oscillates back and forth through the vertical.

**Strategy:** The time the pendulum takes to move from maximum displacement to equilibrium position is one-quarter of a period. Use equation 13-20 to determine the time.

**Solution:** Insert the numeric values into equation 13-20 and convert feet to meters:

$$\frac{T}{4} = \frac{\pi}{2}\sqrt{\frac{L}{g}} = \frac{\pi}{2}\sqrt{\frac{75.0\ \text{ft}}{9.81\ \text{m/s}^2}\left(\frac{0.305\ \text{m}}{\text{ft}}\right)} = \boxed{2.4\ \text{s}}$$

**Insight:** The full period of this pendulum is 4(2.4 s) = 9.6 seconds. A pendulum with only half this length would have a period of 6.8 s.

93. A 0.45-kg crow lands on a slender branch and bobs up and down with a period of 1.5 s. An eagle flies up to the same branch, scaring the crow away, and lands. The eagle now bobs up and down with a period of 4.8 s. Treating the branch as an ideal spring, find **(a)** the effective force constant of the branch and **(b)** the mass of the eagle.

**Picture the Problem**: A crow lands on a branch and it bobs up and down like a mass on a spring. When an eagle lands on the same branch the period of the motion will be slower because the eagle is more massive.

**Strategy:** Use the mass of the crow and the period of oscillation to determine the spring force constant of the branch. Calculate the mass of the eagle from the spring force constant of the branch and the period of the eagle's oscillation.

**Solution: 1. (a)** Solve the period equation for the spring force constant:

$$T = 2\pi\sqrt{\frac{m}{k}} \quad\Rightarrow\quad k = \left(\frac{2\pi}{T}\right)^2 m$$

**2.** Insert the mass and period:

$$k = \left(\frac{2\pi}{1.5\ \text{s}}\right)^2 (0.45\ \text{kg}) = \boxed{7.9\ \text{N/m}}$$

**3. (b)** Solve the period equation for the mass:

$$T = 2\pi\sqrt{\frac{m}{k}} \quad\Rightarrow\quad m = \left(\frac{T}{2\pi}\right)^2 k$$

**4.** Insert the spring force constant and the period:

$$m = \left(\frac{4.8\ \text{s}}{2\pi}\right)^2 (7.9\ \text{N/m}) = \boxed{4.6\ \text{kg}}$$

**Insight:** Even though the amplitudes of oscillation between the crow and the eagle could have been different, they do not affect the period of motion. Therefore it is possible to use the oscillation of the branch in measuring the mass of the eagle.