Discussion Examples
Chapter 9: Linear Momentum and Collisions

3. A 26.2-kg dog is running northward at 2.70 m/s, while a 5.30-kg cat is running eastward at 3.04 m/s. Their 74.0-kg owner has the same momentum as the two pets taken together. Find the direction and magnitude of the owner's velocity.

Picture the Problem: The owner walks slowly toward the northeast while the cat runs eastward and the dog runs northward.

Strategy: Sum the momenta of the dog and cat using the component method. Use the known components of the total momentum to find its magnitude and direction. Let north be in the *y* direction, east in the *x* direction. Use the momentum together with the owner's mass to find the velocity of the owner.

Solution: 1. Use the component method of vector addition to find the owner's momentum:

$$\vec{\mathbf{p}}_{\text{total}} = \vec{\mathbf{p}}_{\text{cat}} + \vec{\mathbf{p}}_{\text{dog}} = m_{\text{cat}}\vec{\mathbf{v}}_{\text{cat}} + m_{\text{dog}}\vec{\mathbf{v}}_{\text{dog}}$$
$$= (5.30\text{ kg})(3.04\text{ m/s }\hat{\mathbf{x}}) + (26.2\text{ kg})(2.70\text{ m/s }\hat{\mathbf{y}})$$
$$\vec{\mathbf{p}}_{\text{total}} = (16.1\text{ kg}\cdot\text{m/s})\hat{\mathbf{x}} + (70.7\text{ kg}\cdot\text{m/s})\hat{\mathbf{y}}$$

2. Divide the owner's momentum by his mass to get the components of the owner's velocity:

$$\vec{\mathbf{p}}_{\text{owner}} = m_{\text{owner}}\vec{\mathbf{v}}_{\text{owner}} = \vec{\mathbf{p}}_{\text{total}}$$
$$\mathbf{v}_{\text{owner}} = \frac{\vec{\mathbf{p}}_{\text{total}}}{m_0} = \frac{(16.1\text{ kg}\cdot\text{m/s})\hat{\mathbf{x}} + (70.7\text{ kg}\cdot\text{m/s})\hat{\mathbf{y}}}{74.0\text{ kg}}$$
$$= (0.218\text{ m/s})\hat{\mathbf{x}} + (0.955\text{ m/s})\hat{\mathbf{y}}$$

3. Use the known components to find the direction and magnitude of the owner's velocity:

$$v_{\text{owner}} = \sqrt{(0.218\text{ m/s})^2 + (0.955\text{ m/s})^2} = \boxed{0.980\text{ m/s}}$$
$$\theta = \tan^{-1}\left(\frac{0.955}{0.218}\right) = \boxed{77.1^\circ}\text{ north of east}$$

Insight: We bent the rules of significant figures a bit in step 3 in order to avoid rounding error. The owner is moving much slower than either the cat or the dog because of his larger mass.

15. A 0.50-kg croquet ball is initially at rest on the grass. When the ball is struck by a mallet, the average force exerted on it is 230 N. If the ball's speed after being struck is 3.2 m/s, how long was the mallet in contact with the ball?

Picture the Problem: The croquet mallet exerts an impulse on the ball, imparting momentum.

Strategy: Find the change in momentum of the croquet ball and then use it to find Δt using equation 9-3.

Solution: Solve equation 9-3 for Δt:

$$\Delta t = \frac{\Delta p}{F_{\text{av}}} = \frac{m(v_{\text{f}} - v_{\text{i}})}{F_{\text{av}}} = \frac{(0.50\text{ kg})(3.2\text{ m/s} - 0)}{230\text{ N}} = 0.0070\text{ s} = \boxed{7.0\text{ ms}}$$

Insight: The large force (52 lb) is exerted over a very brief time to give the ball its small velocity (7.2 mi/h).

25. A 92-kg astronaut and a 1200-kg satellite are at rest relative to the space shuttle. The astronaut pushes on the satellite, giving it a speed of 0.14 m/s directly away from the shuttle. Seven-and-a-half seconds later the astronaut comes into contact with the shuttle. What was the initial distance from the shuttle to the astronaut?

Picture the Problem: The astronaut and the satellite move in opposite directions after the astronaut pushes off. The astronaut travels at constant speed a distance *d* before coming in contact with the space shuttle.

Strategy: As long as there is no friction the total momentum of the astronaut and the satellite must remain zero, as it was before the astronaut pushed off. Use the conservation of momentum to determine the speed of the astronaut, and then multiply the speed by the time to find the distance. Assume the satellite's motion is in the negative *x*-direction.

Solution: 1. Find the speed of the astronaut using conservation of momentum:

$$p_{\text{a}} + p_{\text{s}} = 0 = m_{\text{a}}v_{\text{a}} + m_{\text{s}}v_{\text{s}}$$
$$v_{\text{a}} = -\frac{m_{\text{s}}v_{\text{s}}}{m_{\text{a}}}$$

2. Find the distance to the space shuttle:

$$d = v_{\text{a}}t = -\frac{m_{\text{s}}v_{\text{s}}}{m_{\text{a}}}t = -\frac{(1200\text{ kg})(-0.14\text{ m/s})}{(92\text{ kg})}(7.5\text{ s}) = \boxed{14\text{ m}}$$

Insight: One of the tricky things about spacewalking is that whenever you push on a satellite or anything else, because of Newton's Third Law you yourself get pushed ! Conservation of momentum makes it easy to predict your speed.

34. A 0.430-kg block is attached to a horizontal spring that is at its equilibrium length, and whose force constant is 20.0 N/m. The block rests on a frictionless surface. A 0.0500-kg wad of putty is thrown horizontally at the block, hitting it with a speed of 2.30 m/s and sticking. How far does the putty-block system compress the spring?

Picture the Problem: The putty is thrown horizontally, strikes the side of the block, and sticks to it. The putty and the block move together in the horizontal direction immediately after the collision, compressing the spring.

Strategy: Use conservation of momentum to find the speed of the putty-block conglomerate immediately after the collision, then use equation 7-6 to find the kinetic energy. Use conservation of energy to find the maximum compression of the spring after the collision.

Solution: 1. Set $\vec{\mathbf{p}}_{\rm i} = \vec{\mathbf{p}}_{\rm f}$ and solve for $v_{\rm f}$:

$$m_{\rm p} v_{\rm p} = \left(m_{\rm b} + m_{\rm p}\right) v_{\rm f} \quad \Rightarrow \quad v_{\rm f} = \left(\frac{m_{\rm p}}{m_{\rm b} + m_{\rm p}}\right) v_{\rm p}$$

2. Set $E_{\rm after} = E_{\rm rest}$ after the collision:

$$K_{\rm after} + 0 = 0 + U_{\rm rest}$$

$$\tfrac{1}{2}\left(m_{\rm b} + m_{\rm p}\right)\left(\frac{m_{\rm p}}{m_{\rm b} + m_{\rm p}}\right)^2 v_{\rm p}^2 = \tfrac{1}{2} k x_{\rm max}^2$$

3. Solve the resulting expression for $x_{\rm max}$:

$$x_{\rm max} = \sqrt{\left[\frac{m_{\rm p}^2 v_{\rm p}^2}{k\left(m_{\rm p} + m_{\rm b}\right)}\right]} = \left[\frac{(0.0500\ {\rm kg})^2 (2.30\ {\rm m/s})^2}{(20.0\ {\rm N/m})(0.430 + 0.0500\ {\rm kg})}\right]^2$$

$$= 0.0371\ {\rm m} = \boxed{3.71\ {\rm cm}}$$

Insight: The putty-block conglomerate will compress the spring even farther if $v_{\rm p}$ is larger or if $m_{\rm p}$ is larger.

39. **IP** A charging bull elephant with a mass of 5240 kg comes directly toward you with a speed of 4.55 m/s. You toss a 0.150-kg rubber ball at the elephant with a speed of 7.81 m/s. **(a)** When the ball bounces back toward you, what is its speed? **(b)** How do you account for the fact that the ball's kinetic energy has increased?

Picture the Problem: The ball and the elephant move horizontally toward each other, and then an elastic collision occurs, sending the ball back along the direction it came from.

Strategy: This problem can be very difficult because both the ball and the elephant are moving before and after the collision (see problem 88). However, if we adopt a frame of reference in which the observer is moving with the ball as it approaches the elephant, then in that frame of reference the ball is initially at rest, and equation 9-12 applies. We'll take that approach, finding $\vec{\mathbf{v}}_0$ of the elephant in the ball's frame of reference, determining the ball's final velocity in that frame, and then switching back to the Earth frame of reference to report the ball's final speed. Let the elephant initially travel in the positive direction so $\vec{\mathbf{v}}_{\rm eg} = 4.45\ {\rm m/s}\,\hat{\mathbf{x}}$ and $\vec{\mathbf{v}}_{\rm bg} = -7.91\ {\rm m/s}\,\hat{\mathbf{x}}$.

Solution: 1. (a) Find $\vec{\mathbf{v}}_0$ using equation 3-8:

$$\vec{\mathbf{v}}_{\rm eg,i} = \vec{\mathbf{v}}_{\rm eb,i} + \vec{\mathbf{v}}_{\rm bg,i}$$

$$\vec{\mathbf{v}}_{\rm eb,i} = \vec{\mathbf{v}}_{\rm eg,i} - \vec{\mathbf{v}}_{\rm bg,i} = (4.55\ {\rm m/s})\hat{\mathbf{x}} - (-7.81\ {\rm m/s})\hat{\mathbf{x}} = (12.36\ {\rm m/s})\hat{\mathbf{x}}$$

2. Apply equation 9-12 to find $v_{\rm bb,f}$ for the ball in this frame of reference:

$$v_{\rm bb,f} = \left(\frac{2m_{\rm e}}{m_{\rm e} + m_b}\right) v_{\rm eb,i} = \left[\frac{2(5240\ {\rm kg})}{5240 + 0.150\ {\rm kg}}\right](12.36\ {\rm m/s}) = \underline{\underline{24.7\ {\rm m/s}}}$$

3. Apply equation 3-8 again to find $v_{\rm bg,f}$:

$$\vec{\mathbf{v}}_{\rm bg,f} = \vec{\mathbf{v}}_{\rm bb,f} + \vec{\mathbf{v}}_{\rm bg,i} = (24.7\ {\rm m/s})\hat{\mathbf{x}} + (-7.81\ {\rm m/s})\hat{\mathbf{x}} = (16.9\ {\rm m/s})\hat{\mathbf{x}}$$

$$v_{\rm bg,f} = \boxed{16.9\ {\rm m/s}}$$

4. (b) The ball's kinetic energy has increased because kinetic energy has been transferred from the elephant to the ball as a result of the collision.

Insight: The ball's kinetic energy has increased from 4.57 J to 21.4 J, almost a factor of five! The elephant's speed and kinetic energy hardly changes at all. This is the basic idea behind the gravitational slingshot effect (passage problems 97-100). Note that the approach speed of 12.36 m/s is essentially the same as the separation speed of $16.9 - 4.55 = 12.4$ m/s, as discussed in section 9-6 of the text.

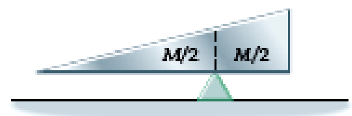
48. **Predict/Explain** A piece of sheet metal of mass *M* is cut into the shape of a right triangle, as shown in **Figure 9–18**. A vertical dashed line is drawn on the sheet at the point where the mass to the left of the line (*M*/2) is equal to the mass to the right of the line (also *M*/2). The sheet is now placed on a fulcrum just under the dashed line and released from rest. **(a)** Does the metal sheet (**A**, remain level; **B**, tip to the left; or **C**, tip to the right)? **(b)** Choose the *best explanation* from among the following:

A. Equal mass on either side will keep the metal sheet level.

B. The metal sheet extends for a greater distance to the left, which shifts the center of mass to the left of the dashed line.

C. The center of mass is to the right of the dashed line because the metal sheet is thicker there.

Picture the Problem: A triangular piece of sheet metal of mass *M* is marked with a vertical dashed line at the point where the mass to the left of the line (*M*/2) is equal to the mass to the right of the line (also *M*/2).



Strategy: Use the definition of the center of mass to answer the question.

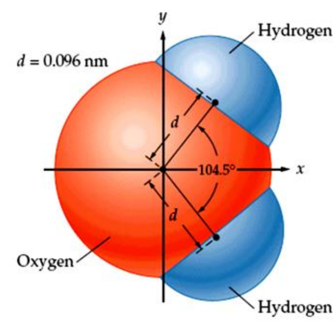
Solution: 1. (a) You can think of the center of mass as the "average" location of the system's mass. The mass *M*/2 on the right, on average, is located closer to the fulcrum than the mass *M*/2 on the left. The center of mass must therefore be to the left of the dashed line, and we conclude that the metal sheet will tip to the left.

2. (b) The best explanation is **II.** The metal sheet extends for a greater distance to the left, which shifts the center of mass to the left of the dashed line. Statements I and III are each false.

Insight: The center of mass of a system is a combination of how much mass it contains and where it is located. If the sheet metal were cut into a rectangular shape, the center of mass would be located right at the dashed line.

83. **The Center of Mass of Water** Find the center of mass of a water molecule, referring to **Figure 9–26** for the relevant angles and distances. The mass of a hydrogen atom is 1.0 u, and the mass of an oxygen atom is 16 u, where u is the atomic mass unit (see Problem 40xx34). Use the center of the oxygen atom as the origin of your coordinate system.

Picture the Problem: The geometry of the water molecule is shown at right.



Strategy: The center of mass of the molecule will lie somewhere along the *x* axis because it is symmetric in the *y* direction. Find X_{cm} using equation 9-14. Both hydrogen atoms will be the same horizontal distance x_{H} from the origin. Let *m* represent the mass of a hydrogen atom, m_{O} the oxygen atom.

Solution: 1. Use equation 9-14 to find X_{cm}

$$X_{\text{cm}} = \frac{\sum mx}{M} = \frac{mx_{\text{H}} + mx_{\text{H}} + m_{\text{O}}x_{\text{O}}}{2m + m_{\text{O}}} = \frac{2my_{\text{O}} + 0}{2m + m_{\text{s}}}$$

$$= \frac{2(1.0\text{ u})(0.096\text{ nm})\cos\left(\tfrac{1}{2}104.5°\right)}{2(1.0\text{ u}) + 16\text{ u}} = \underline{\underline{0.0065\text{ nm}}}$$

2. Recalling that 1 nm = 1×10^{-9} m, we can write $\left(X_{\text{cm}}, Y_{\text{cm}}\right) = \boxed{\left(6.5\times10^{-12}\text{ m},\ 0\right)}$.

Insight: If the angle were to increase from 104.5° the center of mass would move to the left. For instance, if the bond angle were 135° the center of mass would be located at (0, 4.1×10^{-12} m).