

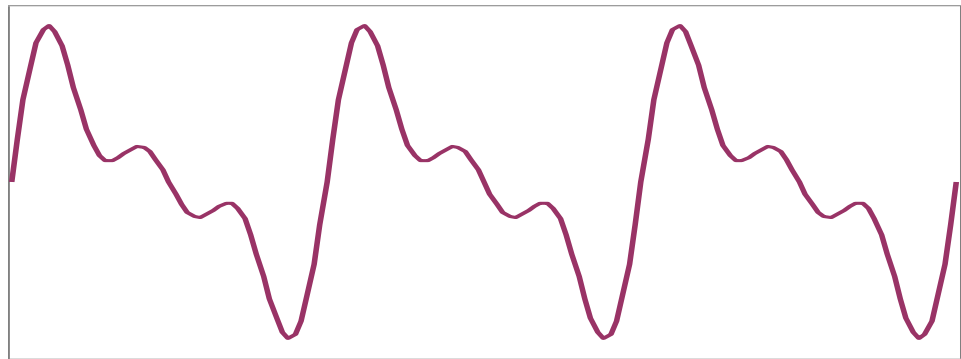
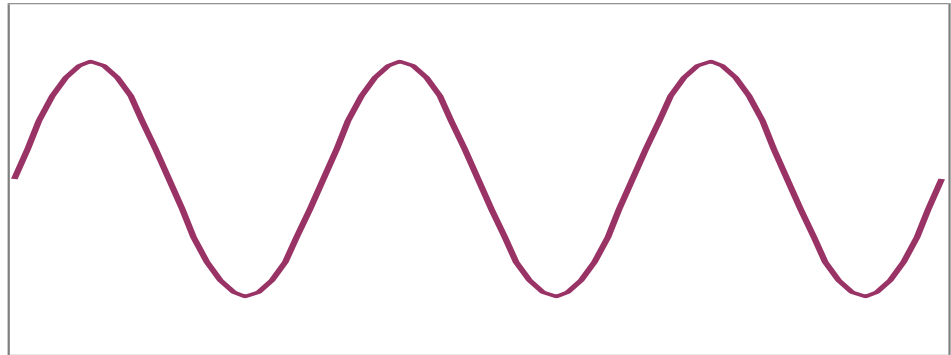
Physics 115 Lecture 13

Fourier Analysis

February 22, 2018

Fourier Synthesis

- **FOURIER SYNTHESIS** is the summing of simple waveforms to create complex waveforms.
- A simple waveform:
- Musical instruments typically produce complex waveforms:



Fourier Synthesis

- Just as with a food recipe, we need to specify HOW MUCH of each ingredient is required.
- We express “how much” by means of a multiplicative factor called a “*FOURIER COEFFICIENT*”:

$$-0.1 \times 1$$

$$-0.7 \times \sin(\omega t)$$

$$0.2 \times \sin(2\omega t)$$

$$0.6 \times \sin(3\omega t)$$

$$-0.3 \times \cos(\omega t)$$

$$0.2 \times \cos(2\omega t)$$

$$0.5 \times \cos(3\omega t)$$

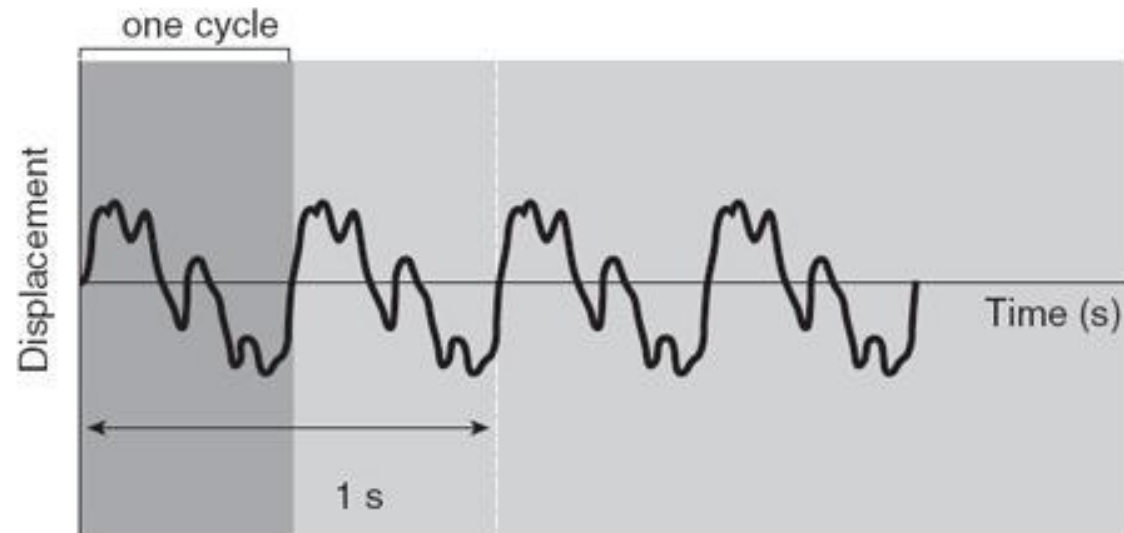
[Interactive applet](#)

[Fourier Wave Game applet](#)

Complex waveforms

- Aperiodic vibration does not have a repeating pattern in time
- Periodic vibration has a characteristic fundamental period and fundamental frequency

$$f_0 = \frac{1}{T_0}$$



Complex waveforms

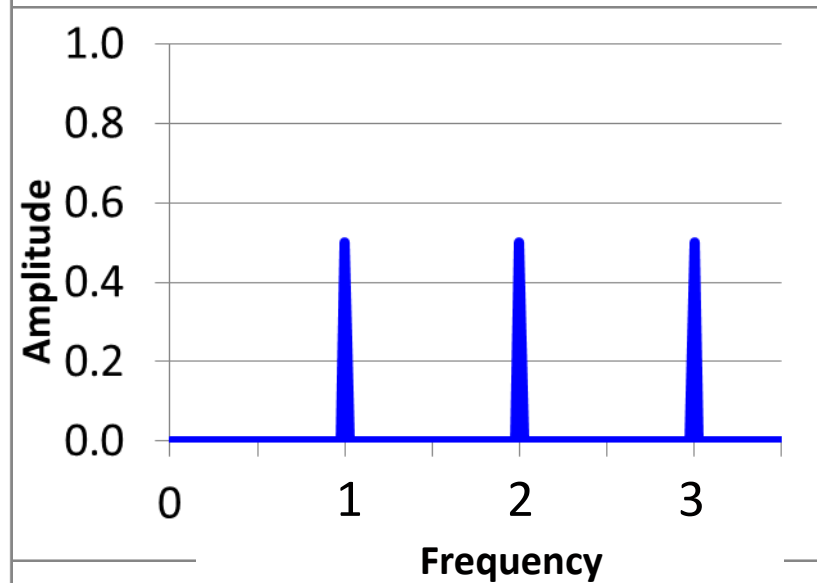
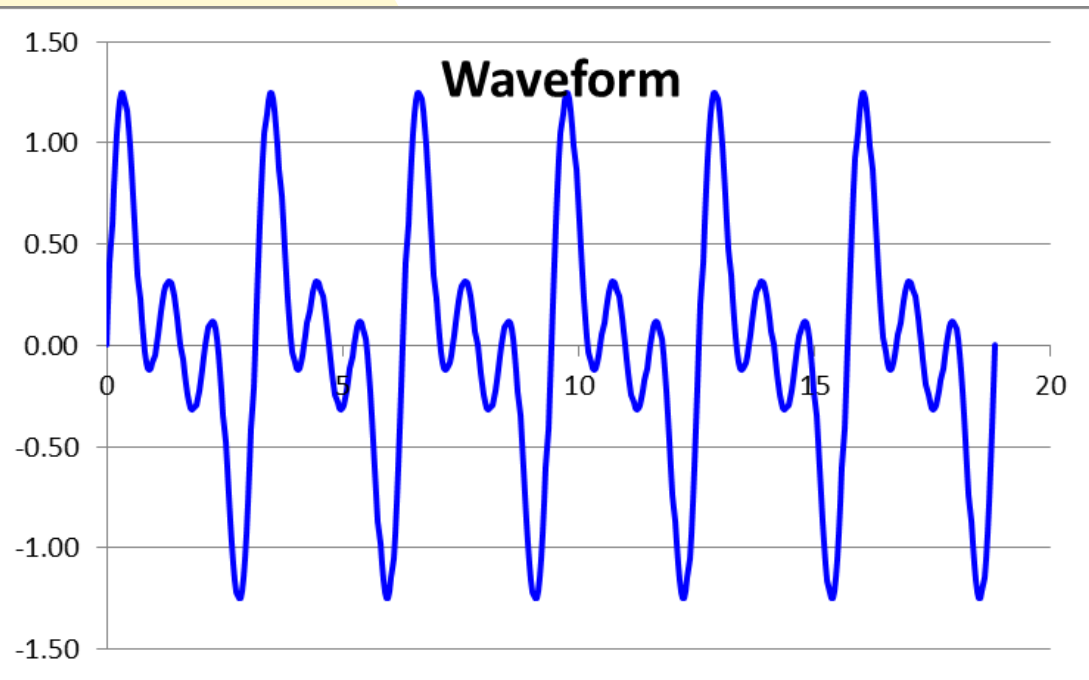
- Aperiodic vibration is composed of frequencies that do not have a common factor among them; that is, it is not composed of frequencies that are integer multiples of a fundamental frequency
- Periodic vibration is composed of frequencies, called *harmonics*, that are integer multiples of a *fundamental frequency*, even if that fundamental frequency is not a component of the vibration (a case of a “missing fundamental”)

Complex waveforms

- The fundamental frequency of a mixture of frequencies is the **greatest common factor** of all the harmonic frequencies that make up the complex waveform.
- For instance, if a complex waveform is made up of frequencies 1500 Hz, 1800 Hz, 2400 Hz, and 2700 Hz, the fundamental frequency is 300 Hz, the greatest common factor among those harmonics.
- The above example is a case of a “missing fundamental” because the complex waveform does not have a 300-Hz component. The waveform is composed of the 5th, 6th, 8th, and 9th harmonics.

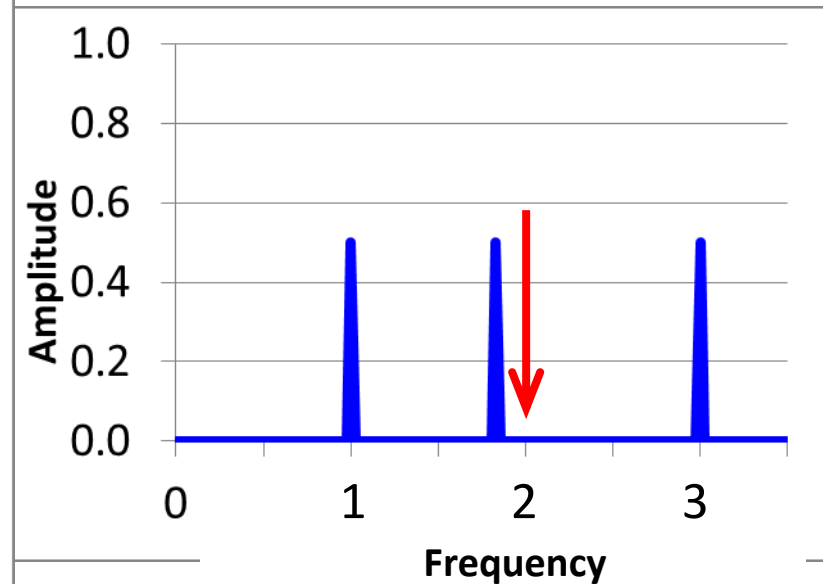
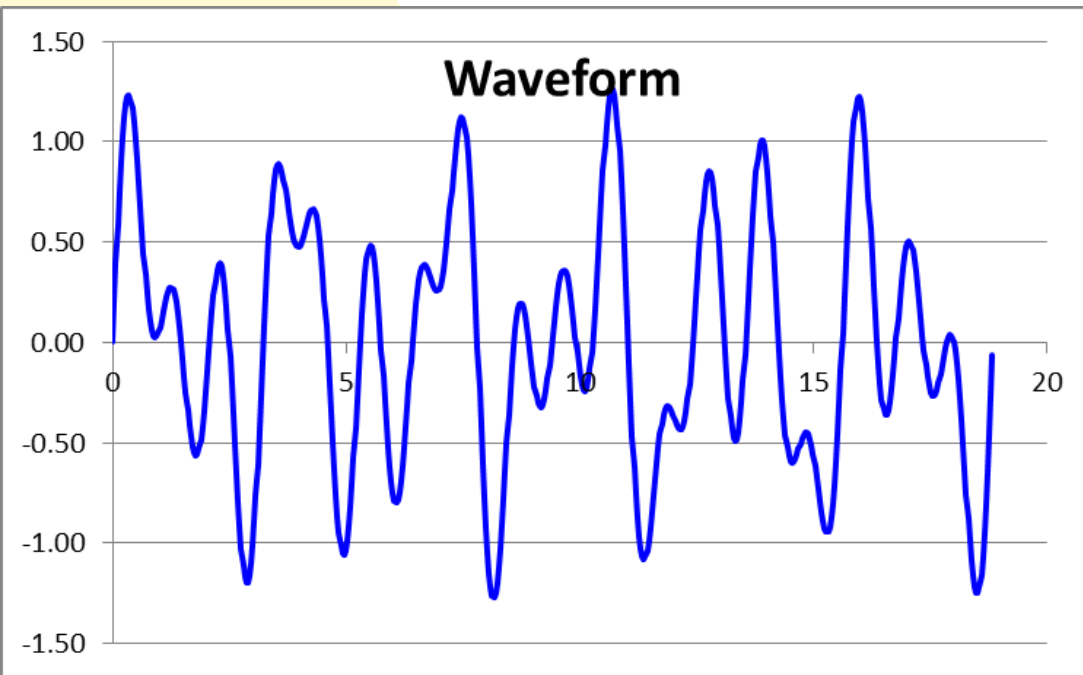
Periodic waveform

$$y = [0.5 \sin(1\omega t)] + [0.5 \sin(2\omega t)] + [0.5 \sin(3\omega t)]$$



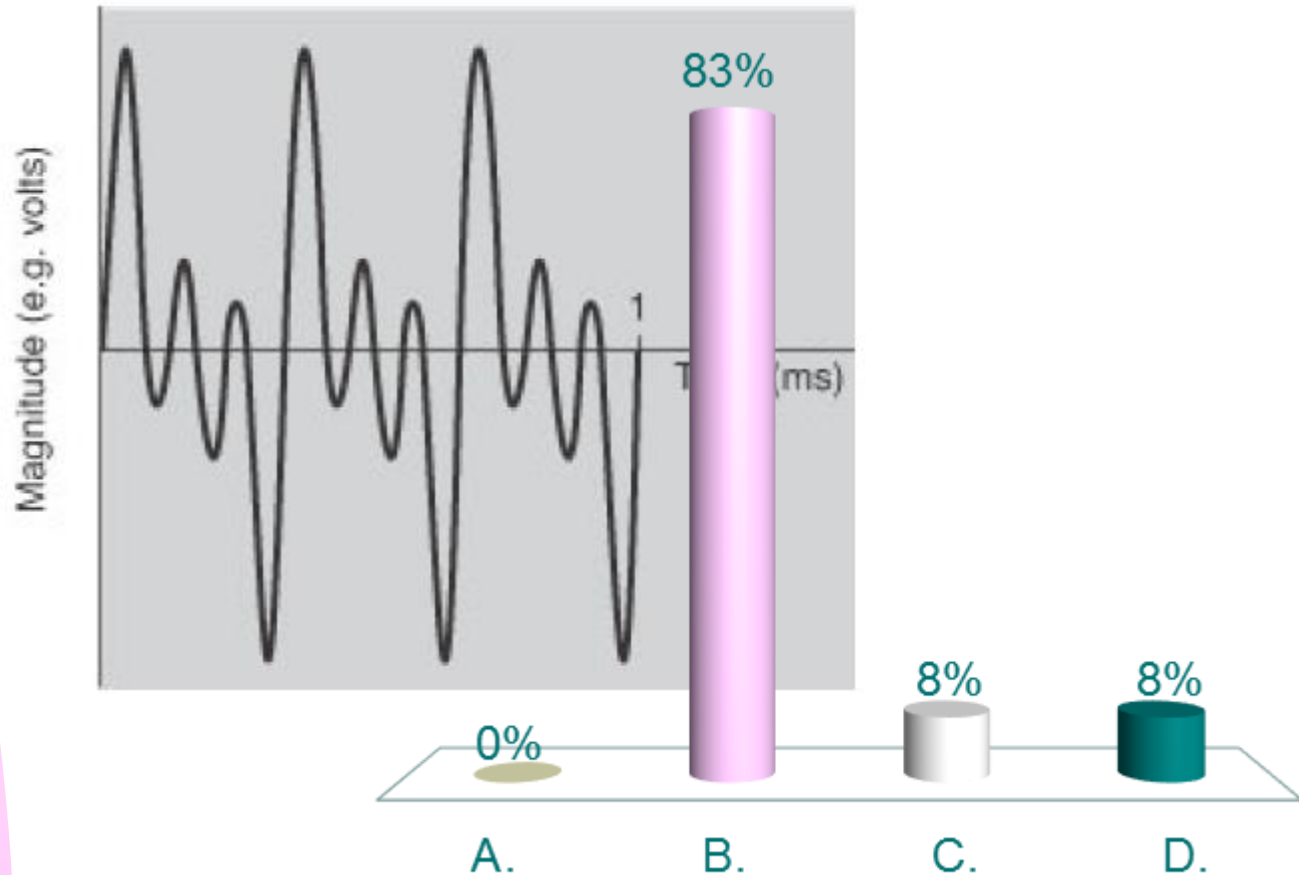
Aperiodic waveform

$$y = [0.5 \sin(1\omega t)] + [0.5 \sin(1.83\omega t)] + [0.5 \sin(3\omega t)]$$



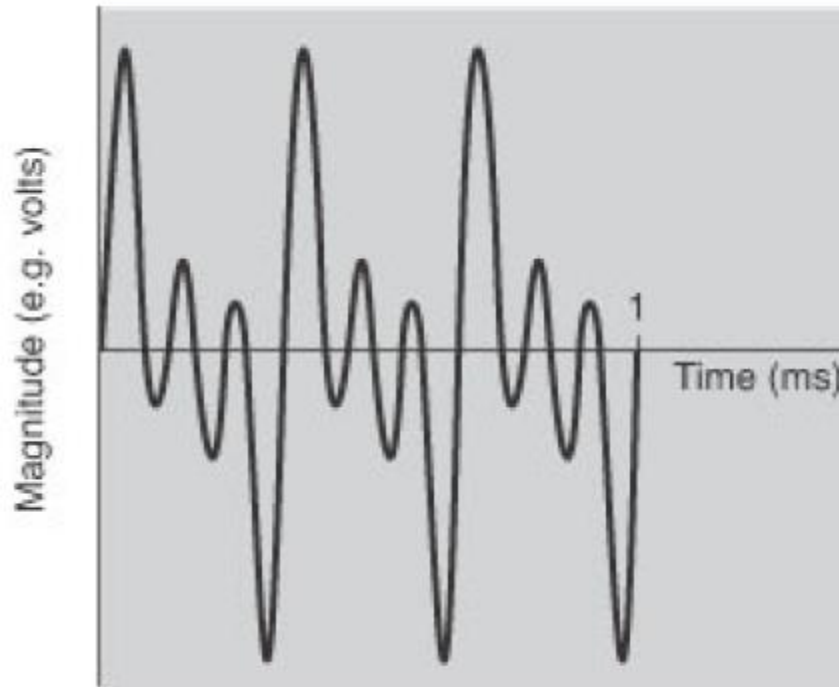
What is the fundamental frequency (f_0) of the waveform?

- A. 1.0 Hz
- B. 333 Hz
- C. 1000 Hz
- D. 3000 Hz



What is the fundamental frequency (f_0) of the waveform?

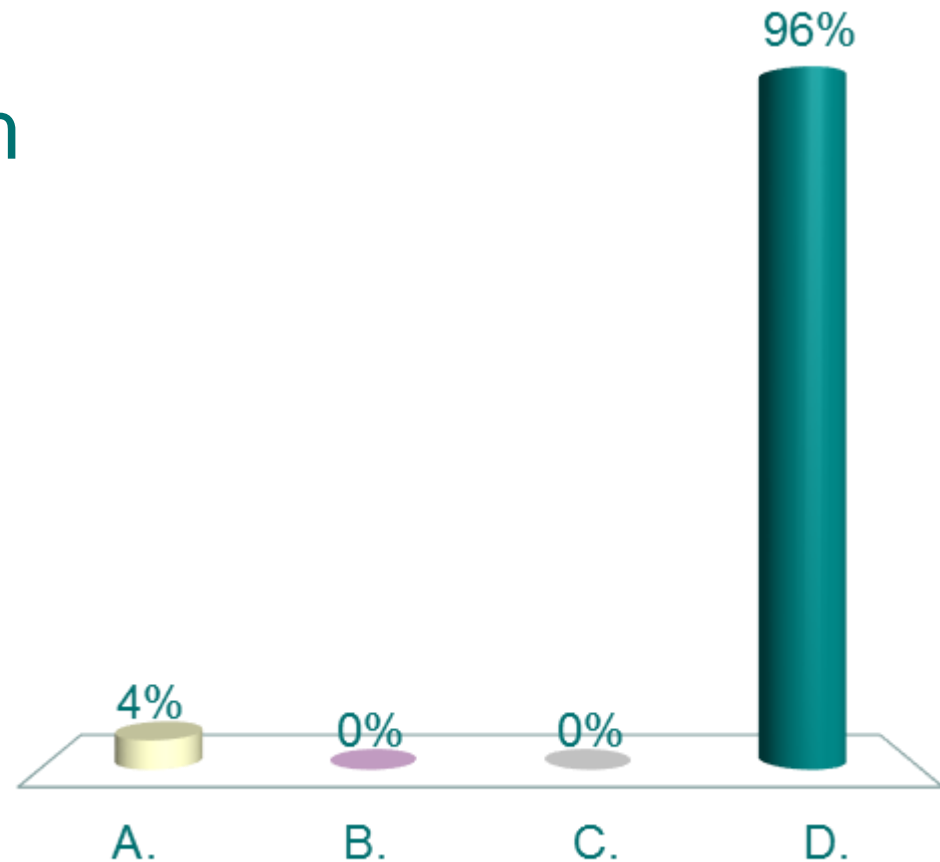
- A. 1.0 Hz
- B. 333 Hz
- C. 1000 Hz
- D. **3000 Hz**



$$f = \frac{3 \text{ cycles}}{1 \text{ ms}} \times \frac{1 \text{ ms}}{1 \times 10^{-3} \text{ s}} = \boxed{3000 \text{ Hz}}$$

If the following components were combined:
150 Hz, 500 Hz, and 675 Hz, what would be the
fundamental frequency?

- A. None; the waveform would be aperiodic
- B. 150 Hz
- C. 50 Hz
- D. 25 Hz



If the following components were combined:
150 Hz, 500 Hz, and 675 Hz, what would be the
fundamental frequency?

A. None; the waveform
would be aperiodic

B. 150 Hz

C. 50 Hz

D. 25 Hz

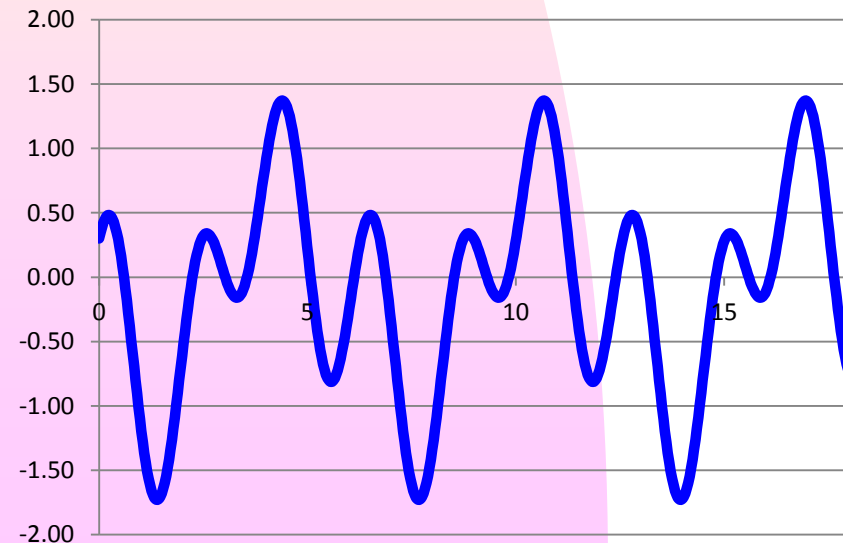
All the frequencies are evenly
divisible by 25 Hz ... but this
would be the case of a “missing
fundamental” (the contributing
frequencies represent the 6th,
20th, and 27th harmonics)

Fourier Synthesis

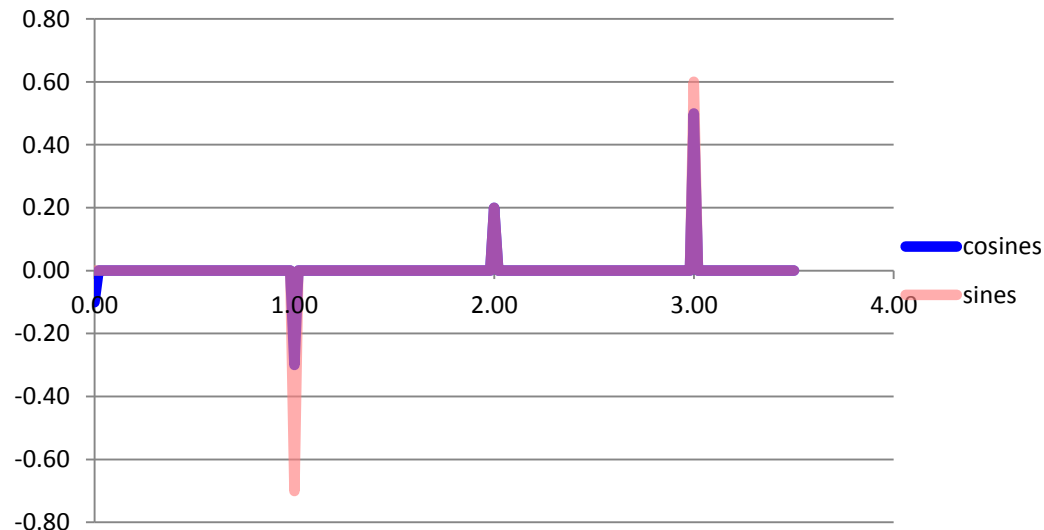
- The complex waveform is obtained by adding the components:

$$y = -0.10 + [-0.7 \sin(\omega t)] + [0.2 \sin(2\omega t)] \\ + [0.6 \sin(3\omega t)] + [-0.3 \cos(\omega t)] \\ + [0.2 \cos(2\omega t)] + [0.5 \cos(3\omega t)]$$

Waveform



Fourier Transform



Fourier Synthesis

- Let's synthesize the following complex waveform using the spreadsheet program EXCEL:

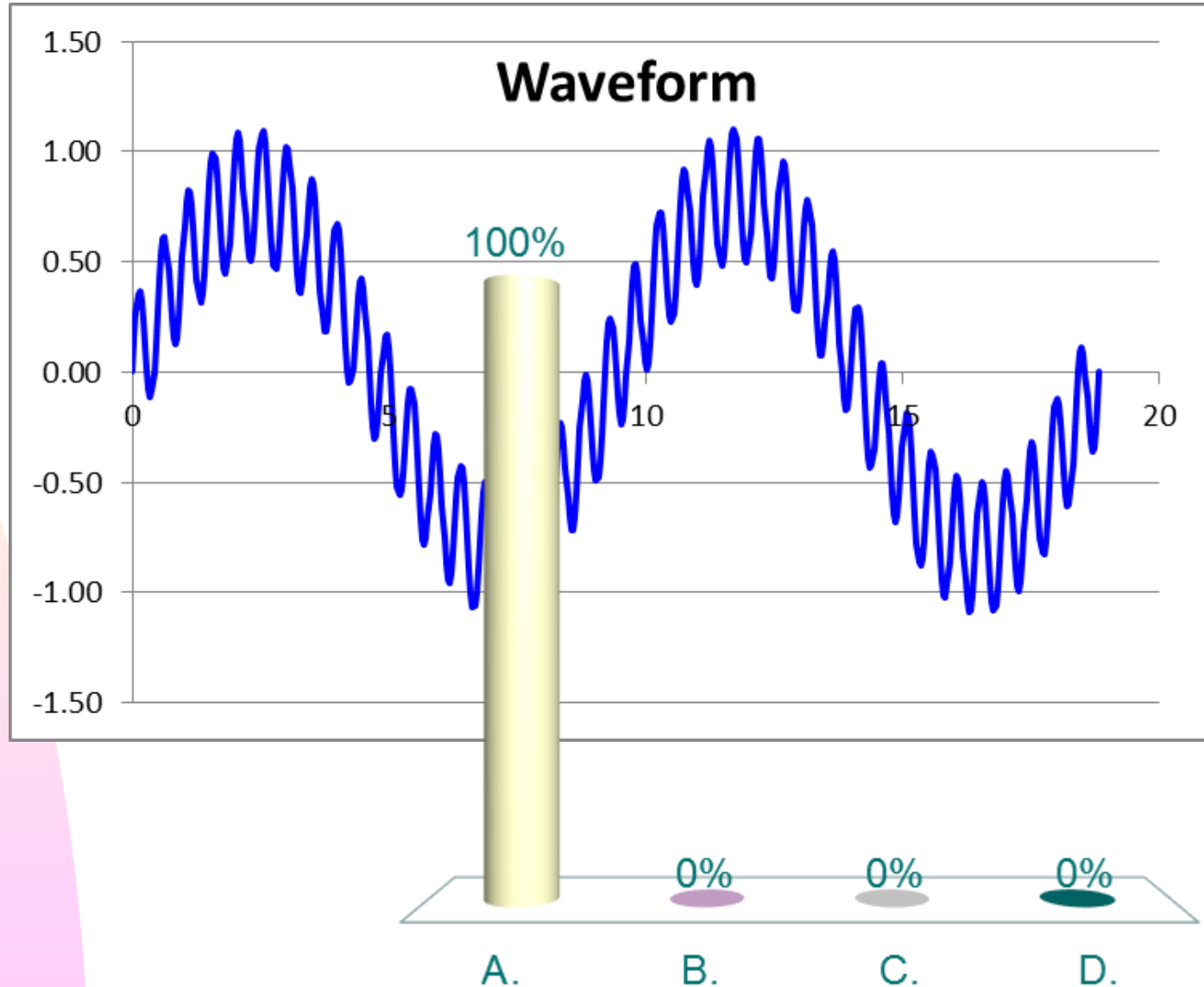
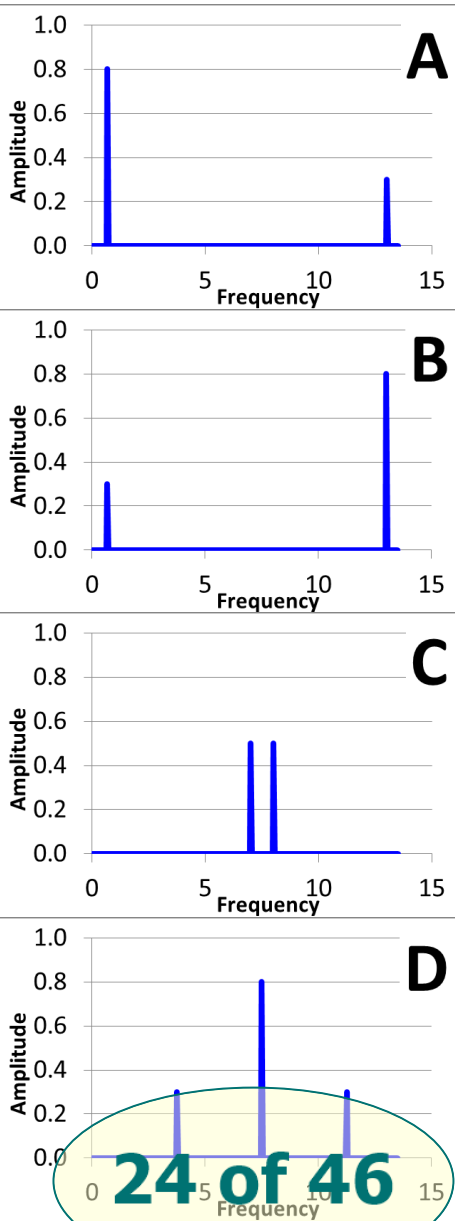
$$y = [1.0 \times \sin(\omega t) + 0.7 \times \sin(2\omega t) + 0.5 \times \sin(3\omega t)]$$

Note: One complete cycle of the complex waveform is obtained by allowing “ ωt ” to range from 0° to 360° .

Fourier Synthesis & Analysis

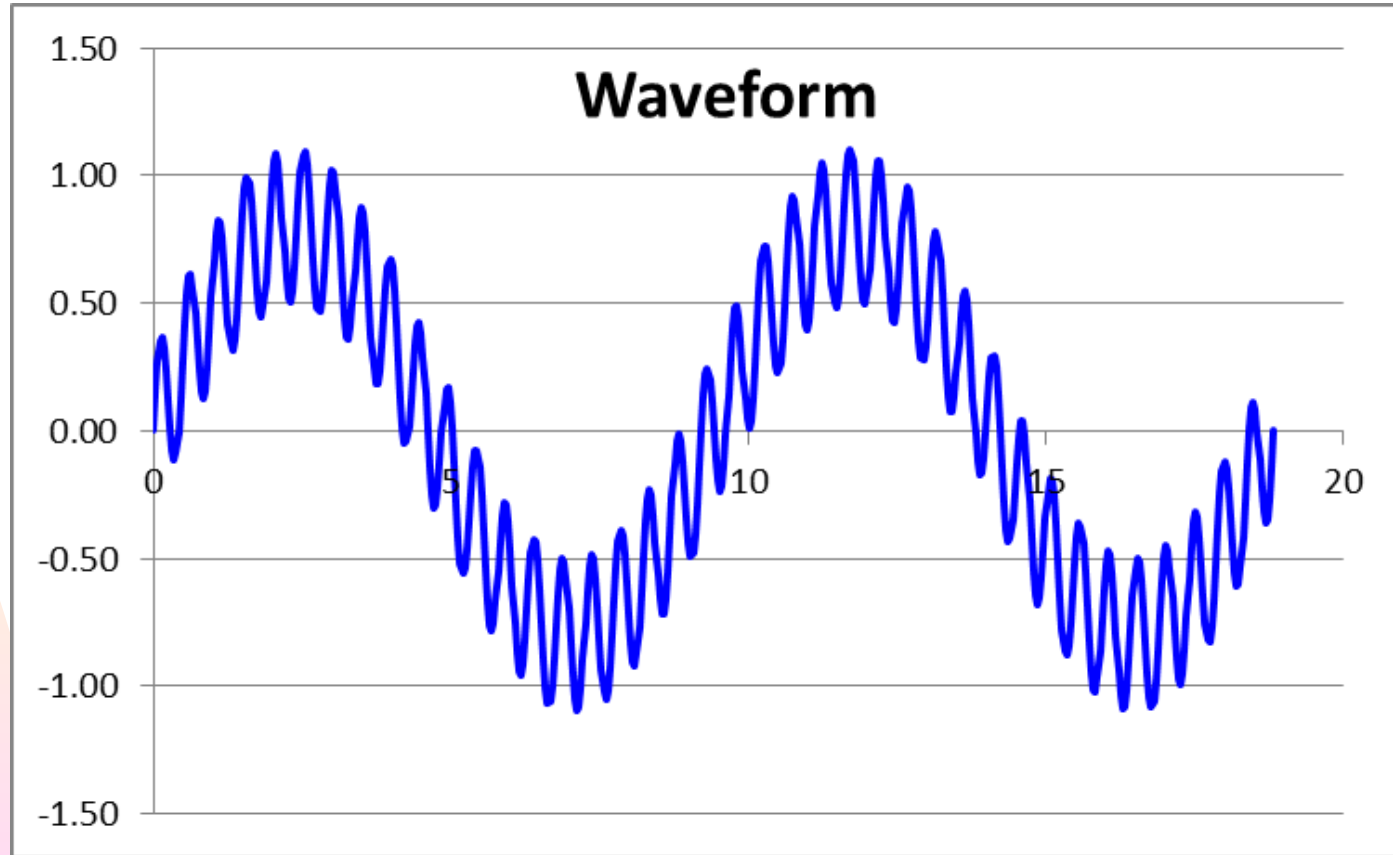
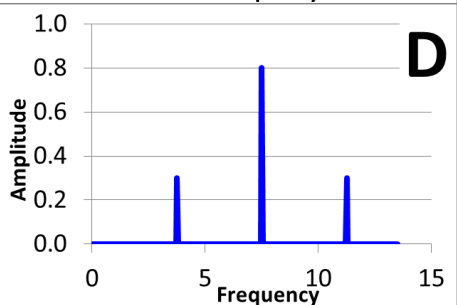
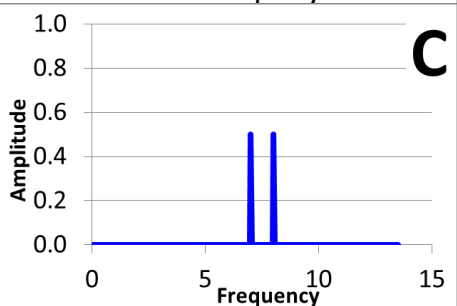
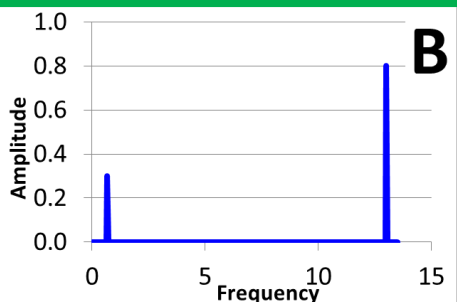
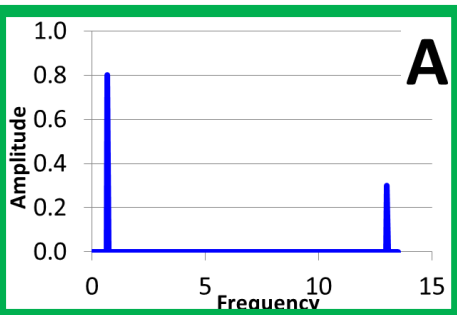
- In FOURIER SYNTHESIS, we create a complex waveform by adding sines and cosines. We can synthesize *ANY* waveform this way.
- Fourier analysis is the opposite of Fourier synthesis. In FOURIER ANALYSIS, we take a complex waveform and break it down into its component sines and cosines! That is, we produce a **Fourier transform** of the waveform.
(We will do this in lab using Microsoft Excel.)

Which graph is the correct frequency spectrum?



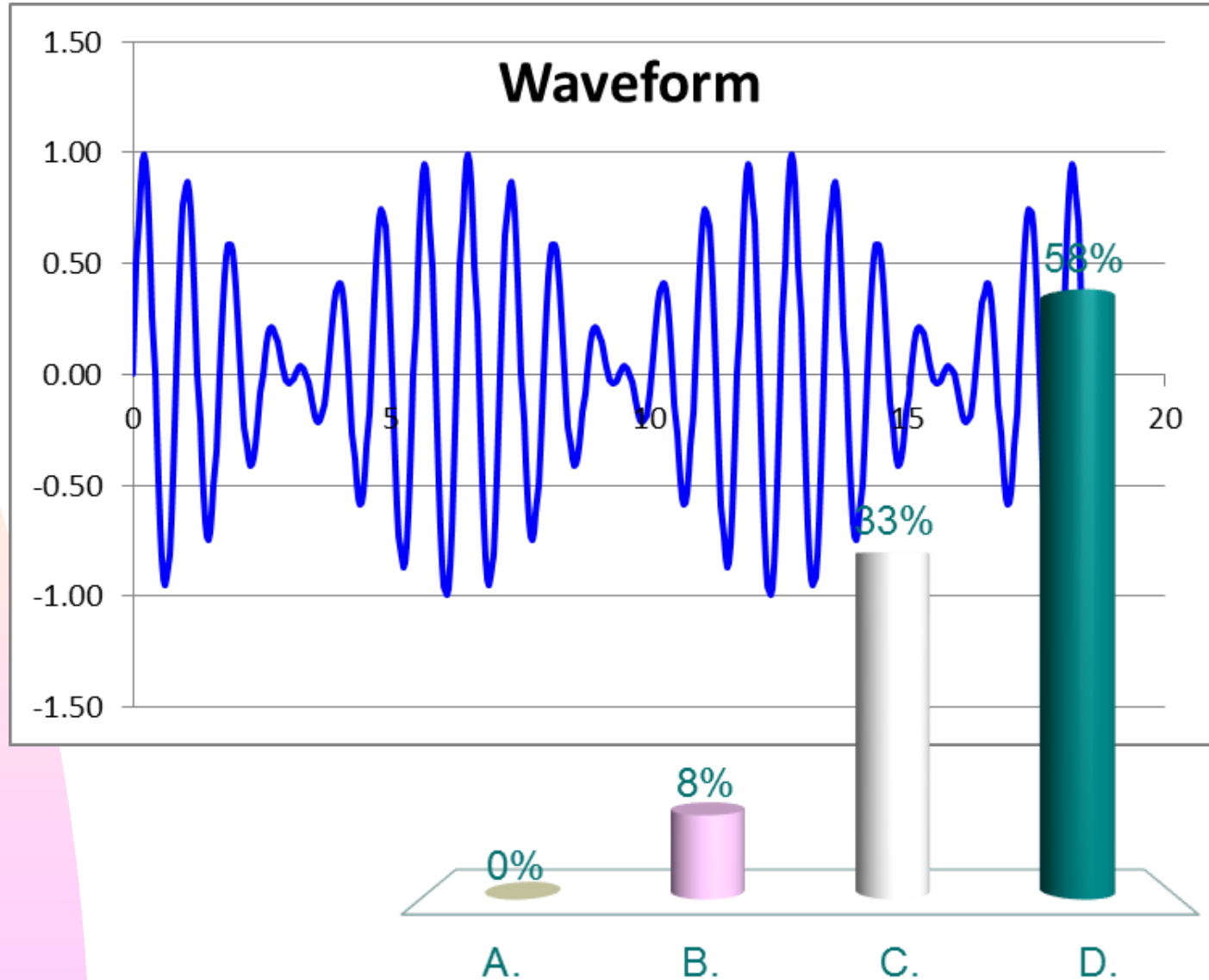
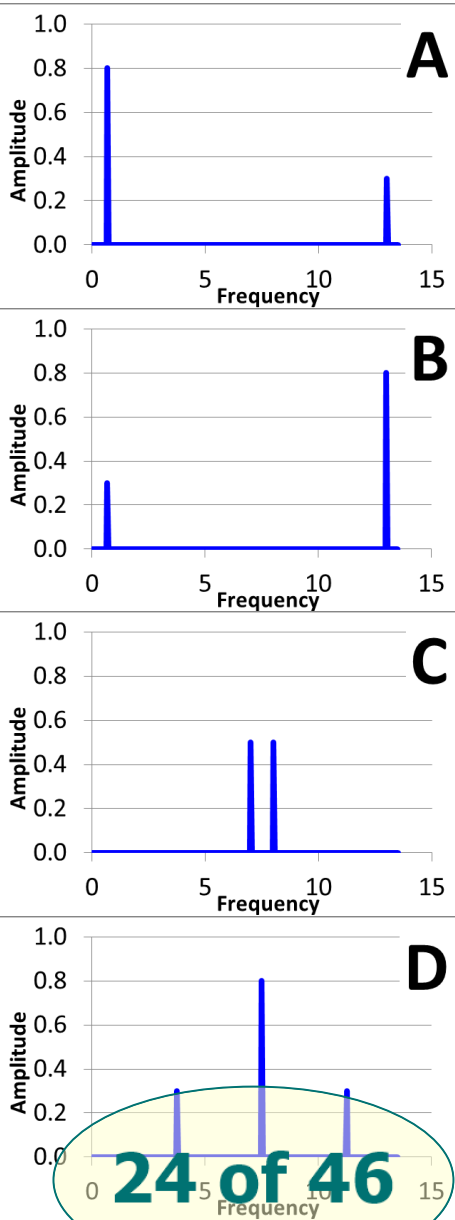
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Which graph is the correct frequency spectrum?



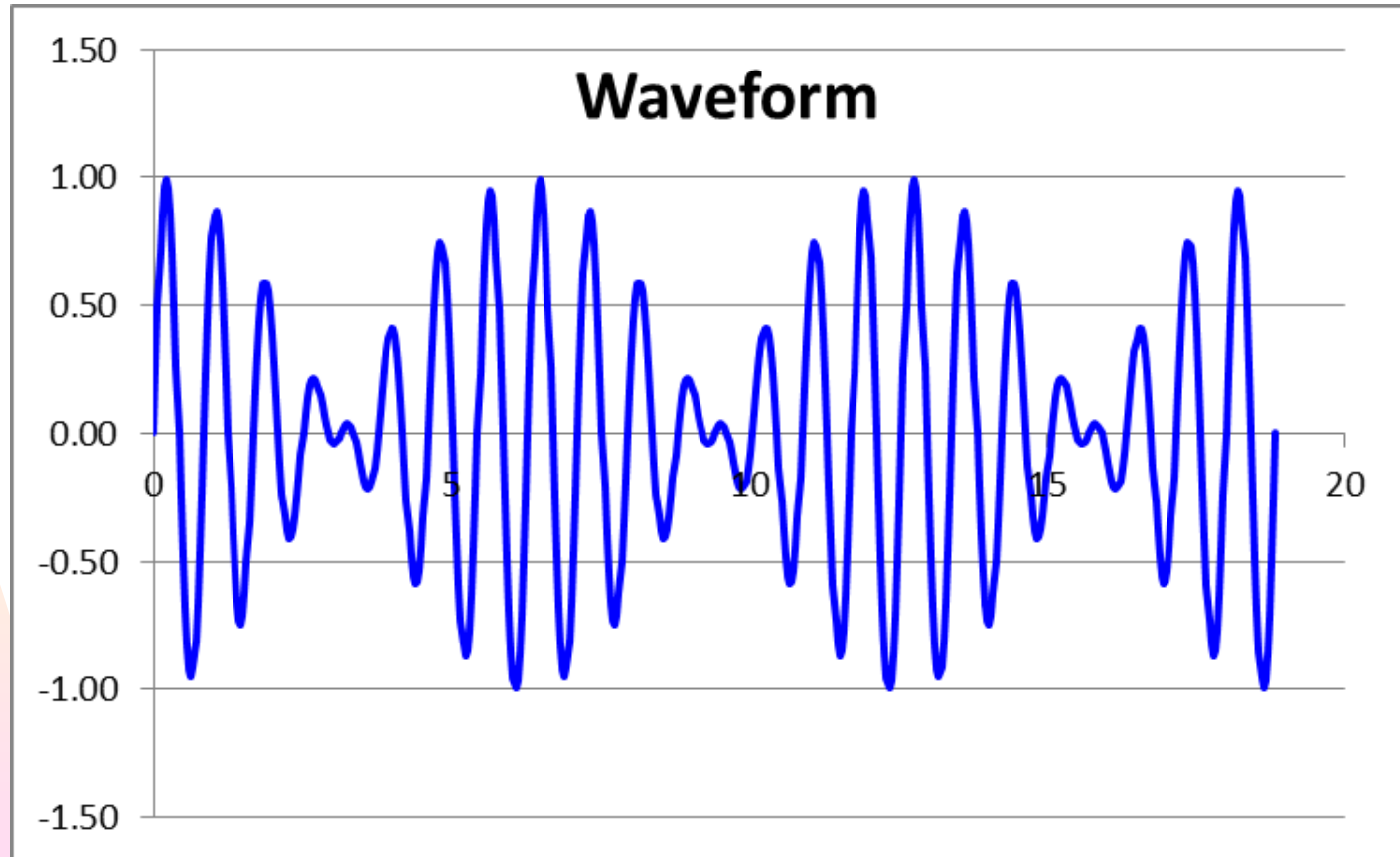
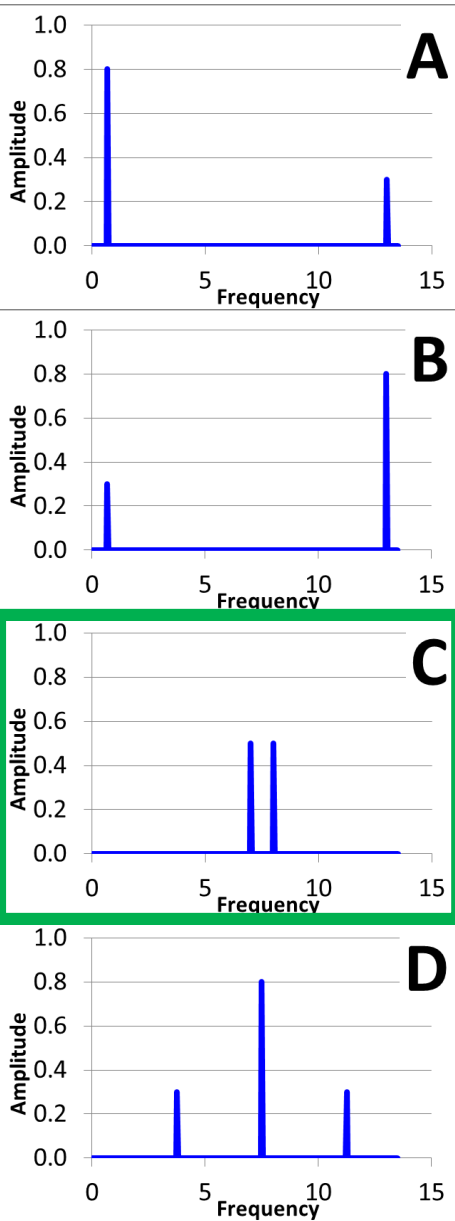
The low-frequency component has a larger amplitude than the high-frequency component

Which graph is the correct frequency spectrum?



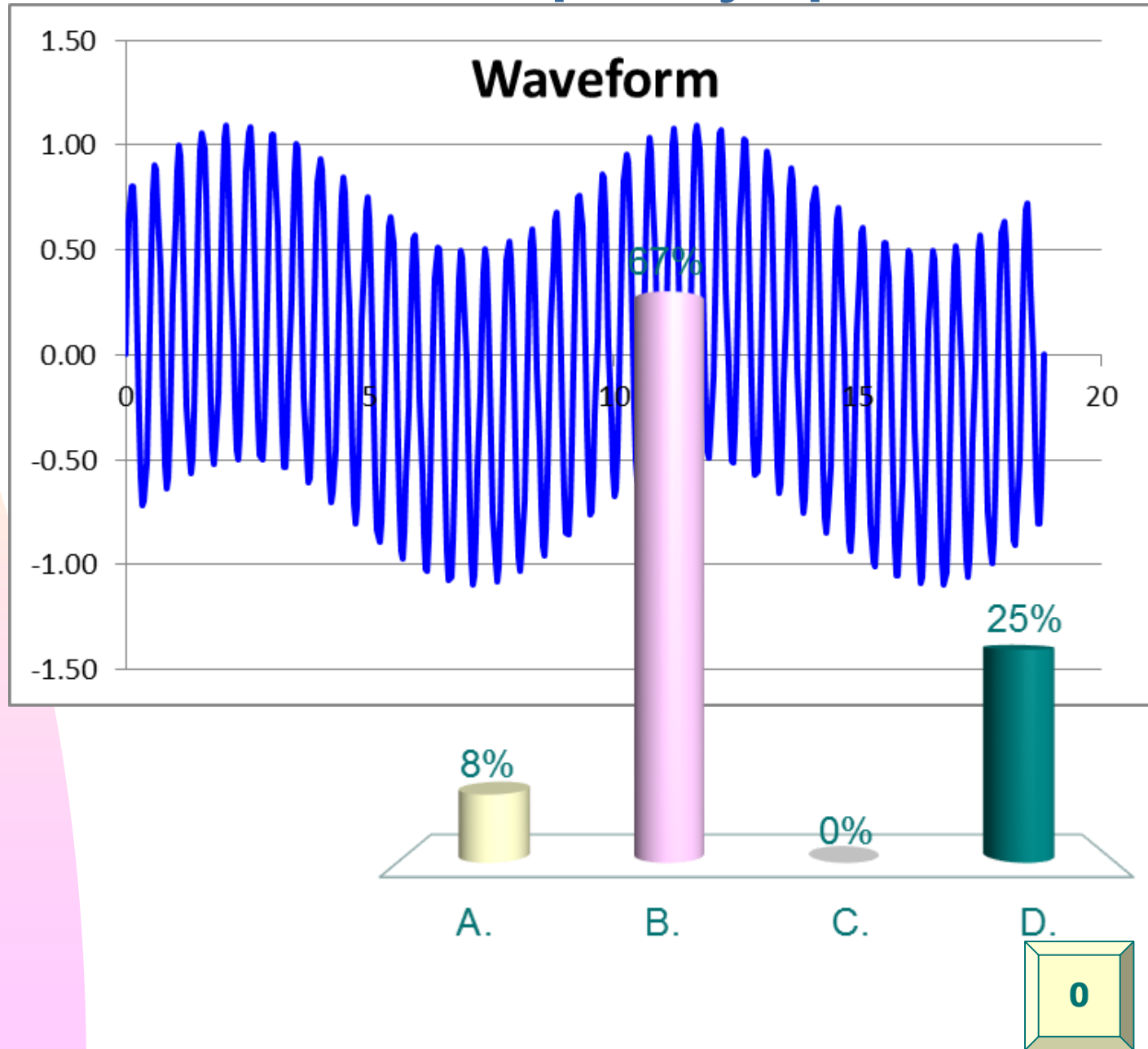
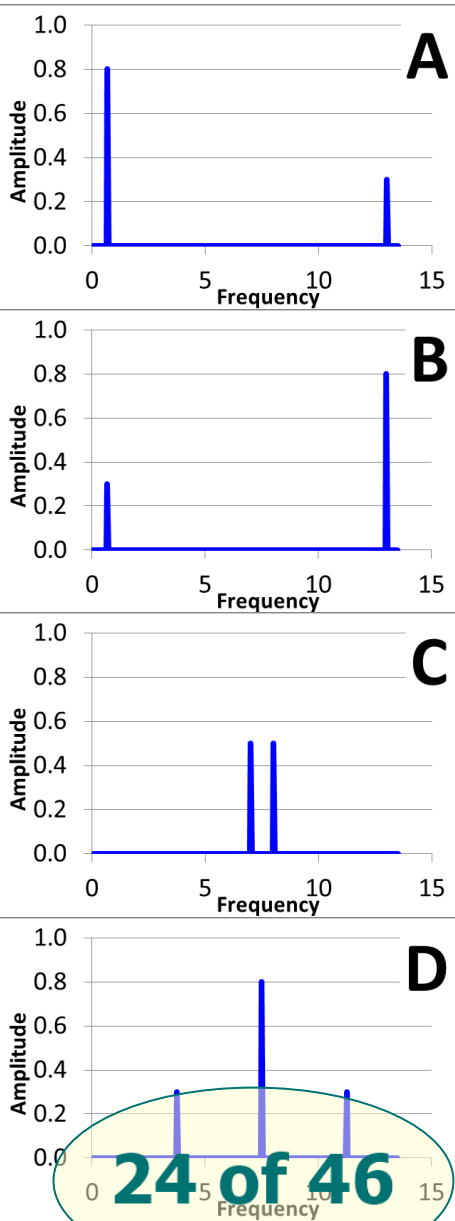
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Which graph is the correct frequency spectrum?



This is a beat pattern, produced by two slightly different frequencies.

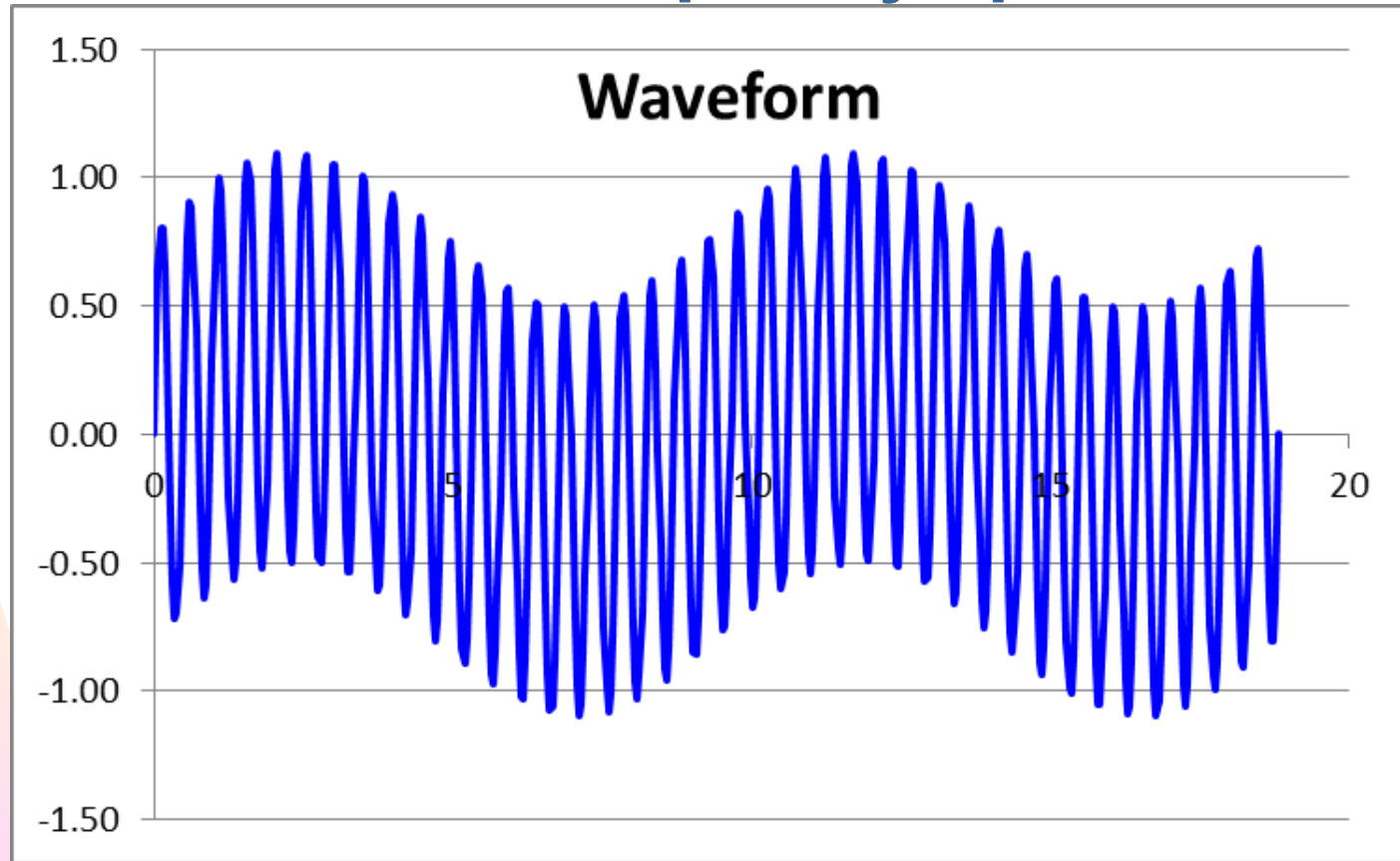
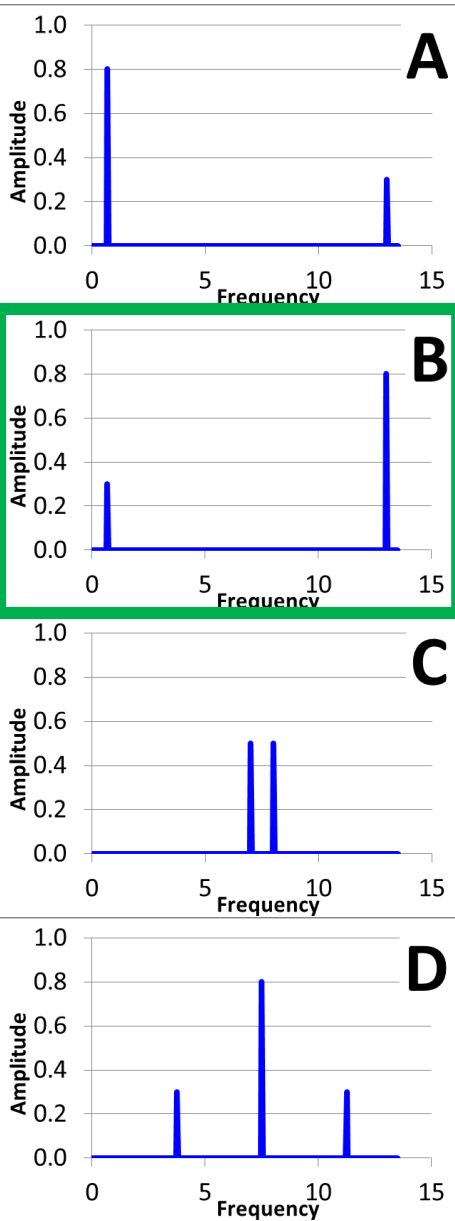
Which graph is the correct frequency spectrum?



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Which graph is the correct frequency spectrum?



The high-frequency component has a larger amplitude than the low-frequency component