

# Physics 115 Lecture 11

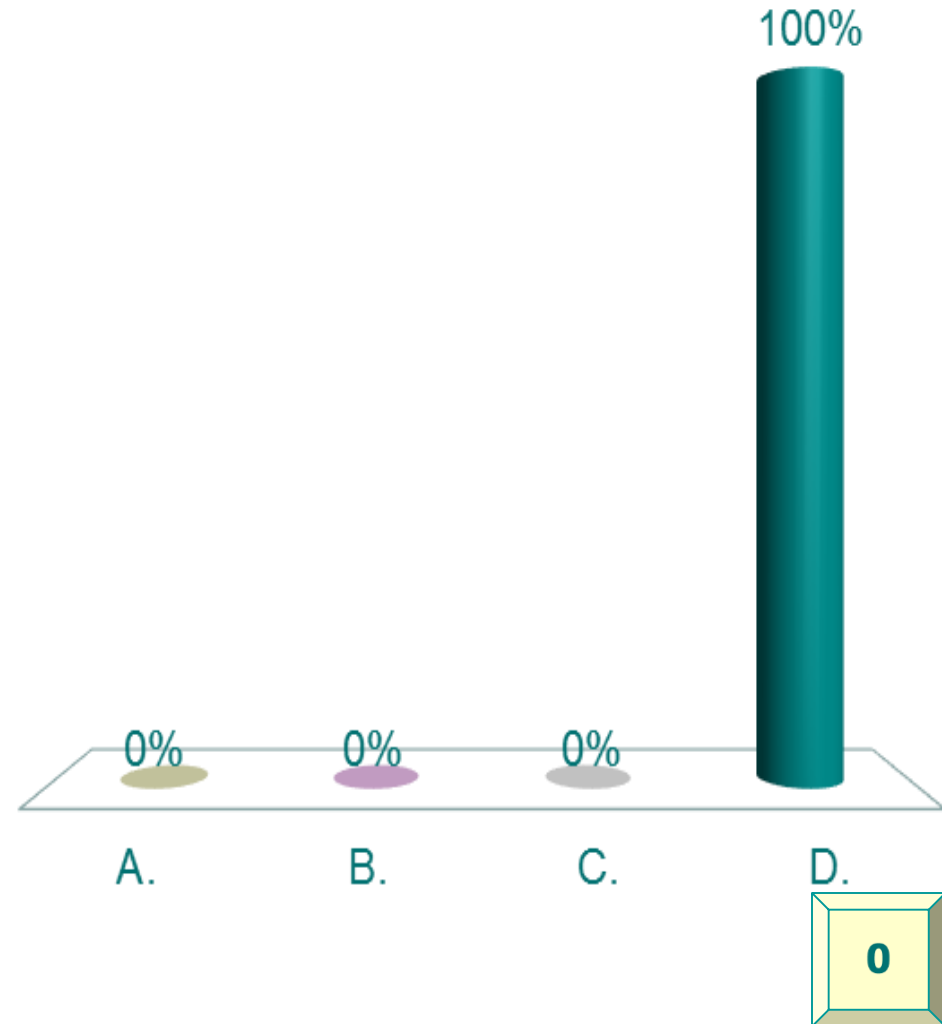
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Fourier Synthesis

**February 15, 2018**

**Class Quiz:** Design an organ pipe, closed at one end and open at the other, that will play the note  $A_6$  (fundamental  $f = 1760$  Hz) when the speed of sound is  $343$  m/s.

- A.  $L = 19.5$  cm
- B.  $L = 14.6$  cm
- C.  $L = 9.8$  cm
- D.  $L = 4.9$  cm



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A.  $L = 19.5$  cm

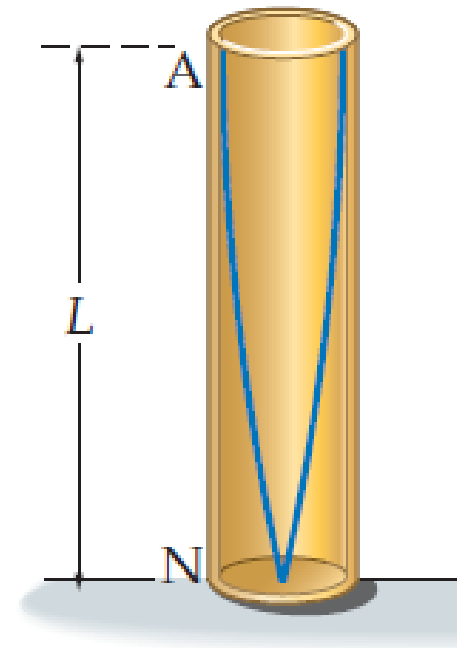
B.  $L = 14.6$  cm

C.  $L = 9.8$  cm

D.  $L = 4.9$  cm

$$L = \frac{\lambda}{4} = \frac{1}{4} \frac{c}{f} = \frac{1}{4} \left( \frac{343 \text{ m/s}}{1760 \text{ Hz}} \right)$$

$$= \frac{1}{4} (0.195 \text{ m}) = 0.049 \text{ m} = \boxed{4.9 \text{ cm}}$$



(a)  
First  
harmonic  
(fundamental)

# Written quiz #3

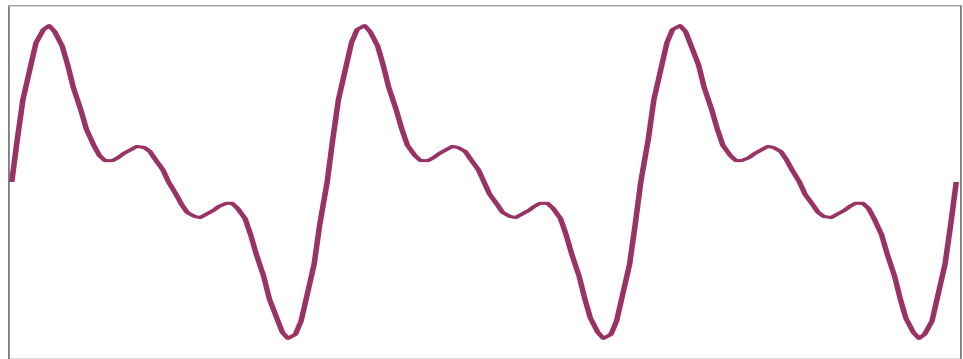
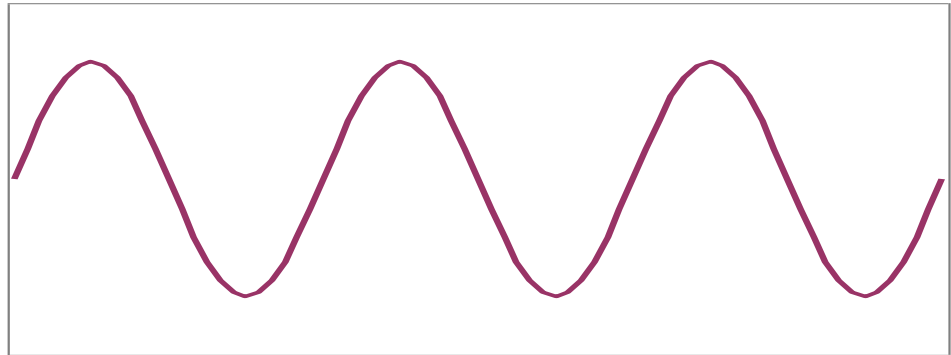
- Based on homework #3
- Posted [answer key](#)

# About the exam

- I will provide the equations (no need to memorize them) and any constants (like density, air pressure, etc.) you may need
- Five numerical problems worth 35 points. Write symbolic formula, show steps leading to answer, present answer with units for full credit
- Ten true/false questions worth 20 points
- Fifteen multiple-choice questions worth 45 points. A few of these may require a simple calculation or ratio

# Fourier Synthesis

- FOURIER SYNTHESIS is the summing of simple waveforms to create complex waveforms.
- A simple waveform:
- Musical instruments typically produce complex waveforms:



# Fourier Synthesis

- Summing simple waveforms to create a complex waveform is like combining simple ingredients to create a complex dish. We need to know which ingredients to use. We also need to know *how much* of each ingredient to use.
- Example:
  - ◆ 1/2 cup      Flour
  - ◆ 2 large      Eggs
  - ◆ 1/4 lb      Butter
  - ◆ 1 cup      Sugar

# Fourier Synthesis

- The ingredients for FOURIER SYNTHESIS are the simple waveforms that are mathematically represented as sines or cosines.
- Ingredients:
  - ◆  $\sin(0\omega t)$   $\cos(0\omega t)$
  - ◆  $\sin(1\omega t)$   $\cos(1\omega t)$
  - ◆  $\sin(2\omega t)$   $\cos(2\omega t)$
  - ◆  $\sin(3\omega t)$   $\cos(3\omega t)$
  - ◆  $\sin(4\omega t)$   $\cos(4\omega t)$
  - ◆ etc. etc.

# Fourier Synthesis

- The ingredients for FOURIER SYNTHESIS are the simple waveforms that are mathematically represented as sines or cosines.
- Ingredients:
  - ◆  ~~$\sin(0\omega t)$~~  → 0       ~~$\cos(0\omega t)$~~  → 1
  - ◆  ~~$\sin(1\omega t)$~~  →  $\sin(\omega t)$        ~~$\cos(1\omega t)$~~  →  $\cos(\omega t)$
  - ◆  $\sin(2\omega t)$        $\cos(2\omega t)$
  - ◆  $\sin(3\omega t)$        $\cos(3\omega t)$
  - ◆  $\sin(4\omega t)$        $\cos(4\omega t)$
  - ◆ etc.      etc.

# Fourier Synthesis

- The ingredients for FOURIER SYNTHESIS are the simple waveforms that are mathematically represented as sines or cosines.

- Ingredients:

NEGLECT THE ZERO

|   |                                                            |                                                            |
|---|------------------------------------------------------------|------------------------------------------------------------|
| ◆ | <del><math>\sin(0\omega t)</math></del> → 0                | <del><math>\cos(0\omega t)</math></del> → 1                |
| ◆ | <del><math>\sin(1\omega t)</math></del> → $\sin(\omega t)$ | <del><math>\cos(1\omega t)</math></del> → $\cos(\omega t)$ |
| ◆ | $\sin(2\omega t)$                                          | $\cos(2\omega t)$                                          |
| ◆ | $\sin(3\omega t)$                                          | $\cos(3\omega t)$                                          |
| ◆ | $\sin(4\omega t)$                                          | $\cos(4\omega t)$                                          |
| ◆ | etc.                                                       | etc.                                                       |

# Fourier Synthesis

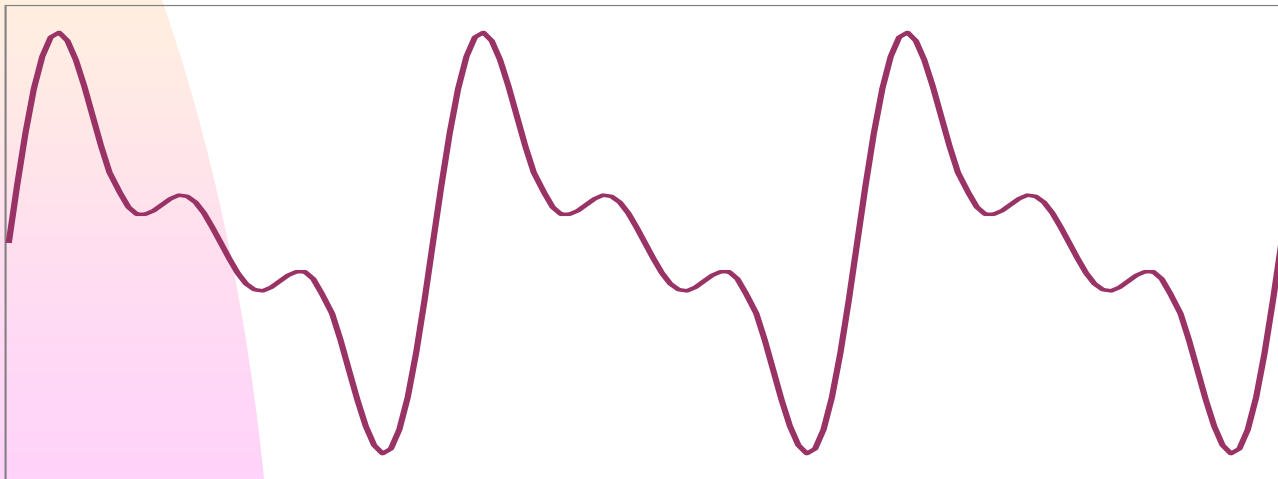
- The ingredients for FOURIER SYNTHESIS are the simple waveforms that are mathematically represented as sines or cosines.
- Ingredients:
  - ◆ 1
  - ◆  $\sin(\omega t)$   $\cos(\omega t)$
  - ◆  $\sin(2\omega t)$   $\cos(2\omega t)$
  - ◆  $\sin(3\omega t)$   $\cos(3\omega t)$
  - ◆  $\sin(4\omega t)$   $\cos(4\omega t)$
  - ◆ etc. etc.

# Fourier Synthesis

- What is “ $\omega$ ”?

$$\omega = \frac{360 \text{ degrees}}{\text{repeat time}}$$

**REPEAT TIME**



# Fourier Synthesis

- Just as with a food recipe, we need to specify HOW MUCH of each ingredient is required.
- We express “how much” by means of a multiplicative factor called a “*FOURIER COEFFICIENT*”:

$$0.8 \times 1$$

$$1.0 \times \sin(\omega t)$$

$$0.7 \times \sin(2\omega t)$$

$$0.5 \times \sin(3\omega t)$$

$$- 0.4 \times \cos(\omega t)$$

$$1.3 \times \cos(2\omega t)$$

$$- 0.1 \times \cos(3\omega t)$$

Interactive  
applet