Physics 115 Lecture 3

Oscillations and energy

January 26, 2018

Simple Harmonic Motion

Period = Time (in seconds) for one complete cycle.
For a spring-and-mass system:

$$T = 2\pi\sqrt{\frac{m}{k}}$$

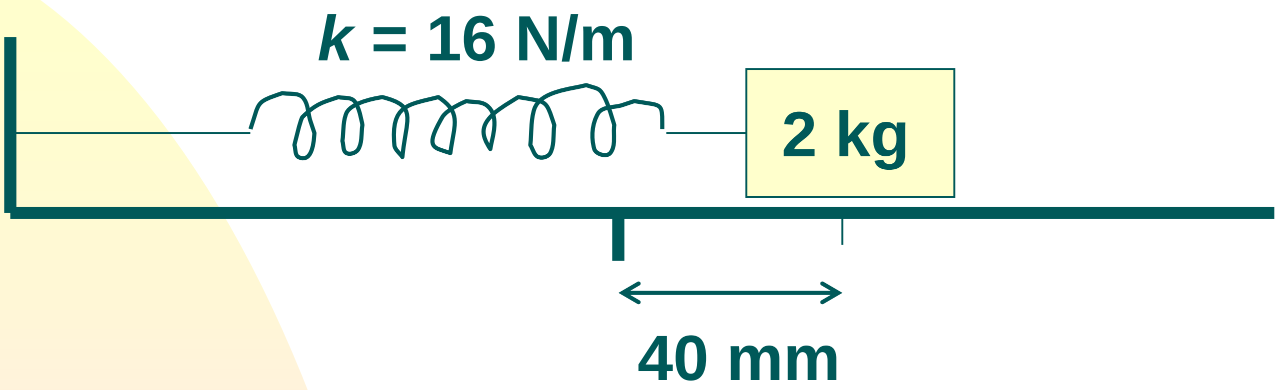
T = period (in seconds)
m = mass (in kg)
k = spring constant (in N/m)

Once the period has been calculated, two other important quantities can be calculated:

Frequency $f = 1/T \leftarrow$ *f* is in Hz if *T* is in seconds

Angular speed $\omega = 360°/T \leftarrow \omega$ is in deg/s if *T* is in seconds

Find the frequency at which this system oscillates:

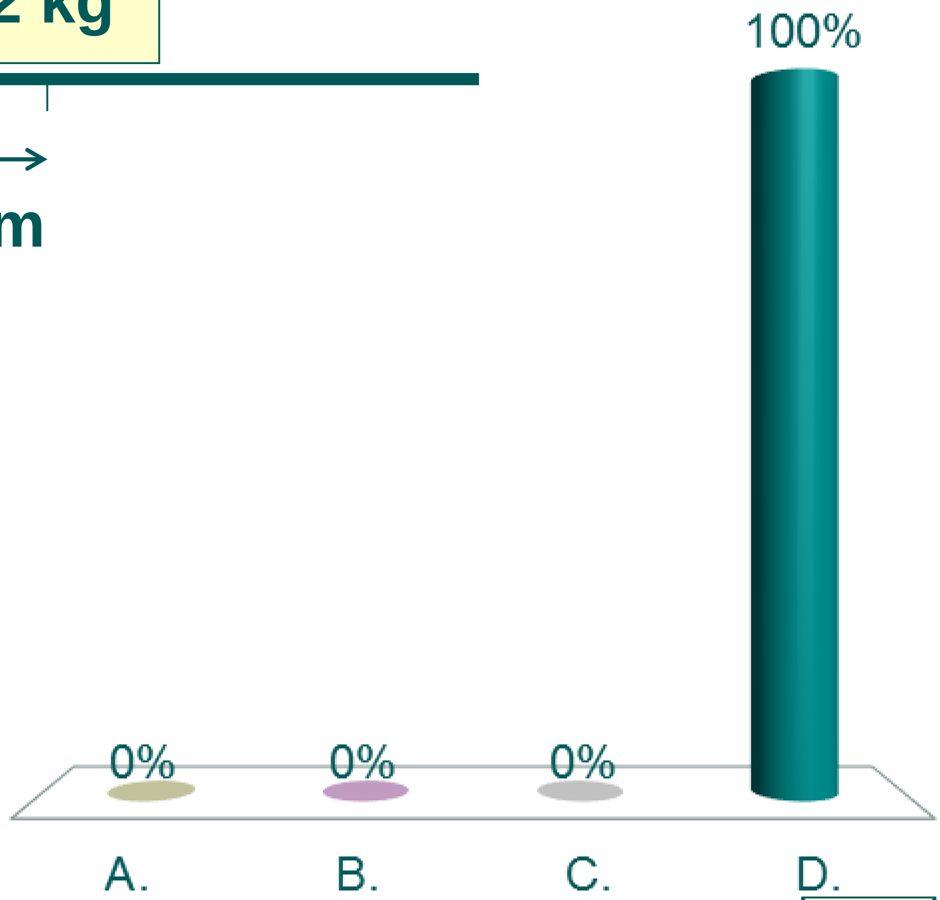


A. 2.2 Hz

B. 8.0 Hz

C. 0.040 Hz

D. 0.450 Hz



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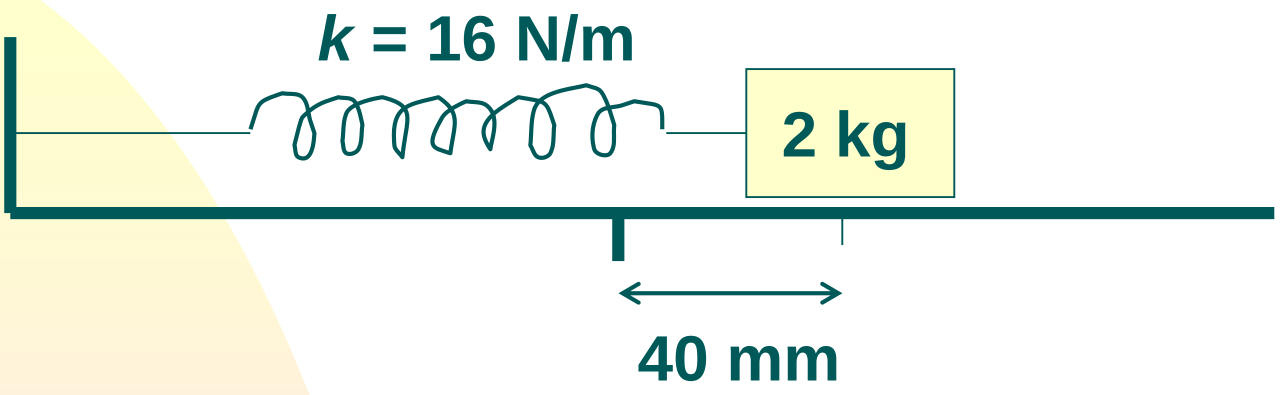
Find the frequency at which this system oscillates:



A. 2.2 Hz

B. 8.0 Hz

C. 0.040 Hz

D. 0.450 Hz

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{2\text{ kg}}{16\text{ N/m}}} = 2.22\text{ s}$$

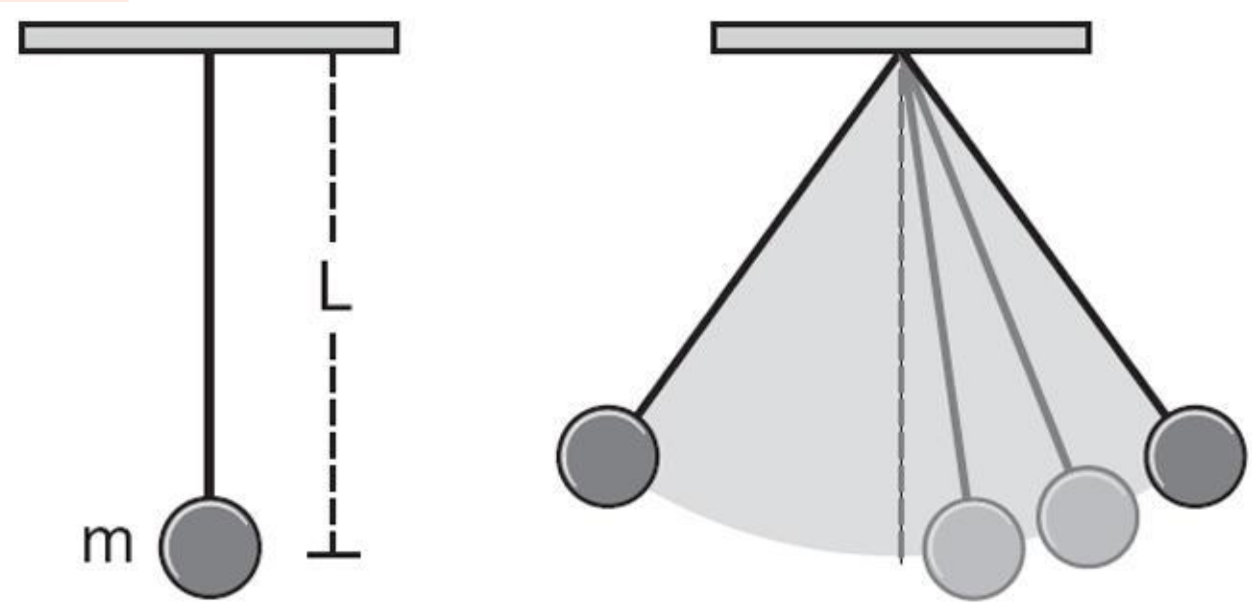
$$f = \frac{1}{T} = \frac{1}{2.22\text{ s}} = \boxed{0.450\text{ Hz}}$$

Simple Harmonic Motion

For a pendulum:

$$T = 2\pi\sqrt{\frac{L}{g}}$$

T = period (in seconds)
L = length of pendulum (in m)
g = accel of gravity (in m/s^2)



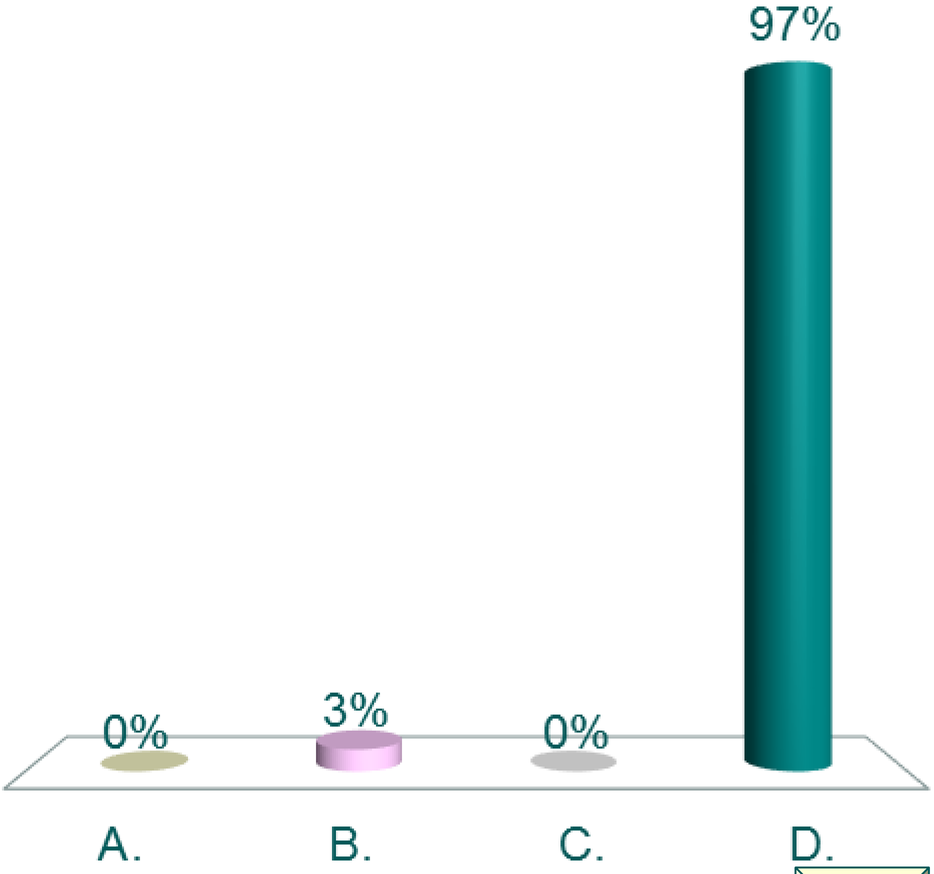
What length pendulum will have a period of 10.0 s? Assume $g = 9.80\ m/s^2$.

A. 10.0 m

B. 980 m

C. 0.629 m

D. 24.8 m



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What length pendulum will have a period of 10.0 s? Assume g = 9.80 m/s^2.

A. 10.0 m

B. 980 m

C. 0.629 m

D. 24.8 m

$$T = 2\pi\sqrt{\frac{L}{g}} \Rightarrow T^2 = 4\pi^2\frac{L}{g}$$

$$\frac{gT^2}{4\pi^2} = L = \frac{\left(9.80\ \text{m/s}^2\right)\left(10.0\ \text{s}\right)^2}{4\pi^2} = \boxed{24.8\ \text{m}}$$

Kinetic Energy

Any mass that is in motion has kinetic energy. The amount of kinetic energy it has is given by:

$$KE = \frac{1}{2} m v^2$$

m = the mass of the moving object, in kilograms.

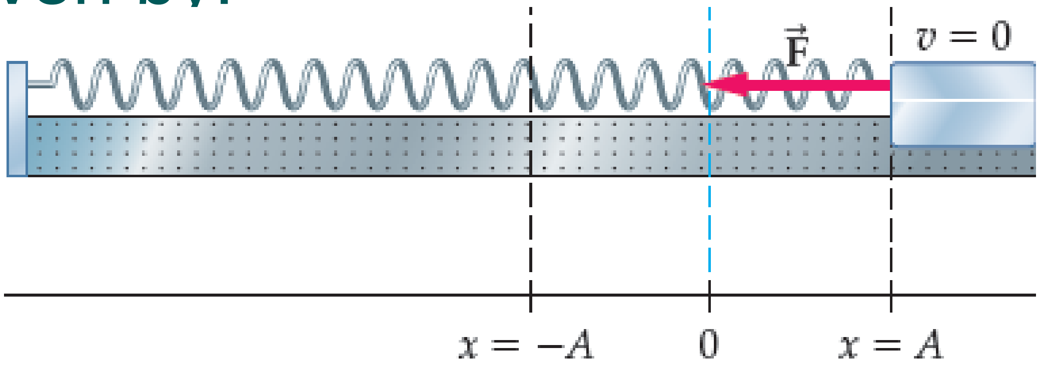
v = the speed of the object, in meters per second.

KE = kinetic energy, in joules (J).

Potential Energy

Any spring that is stretched or compressed has <u>potential energy</u>. The amount of potential energy it has is given by:

$$PE = \frac{1}{2} k x^2$$



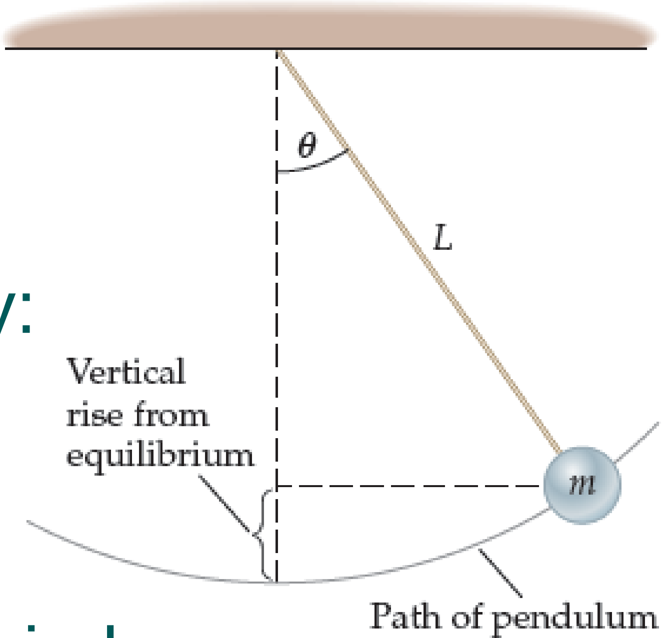
k = the spring constant, in newtons per meter.

x = amount of stretch or compression of spring, in meters.

PE = potential energy, in joules.

Potential Energy

Any pendulum that is pulled away from its equilibrium position has potential energy. The amount of potential energy it has is given by:



$$PE = m\,g\,h$$

m = the mass of the pendulum bob, in kg.

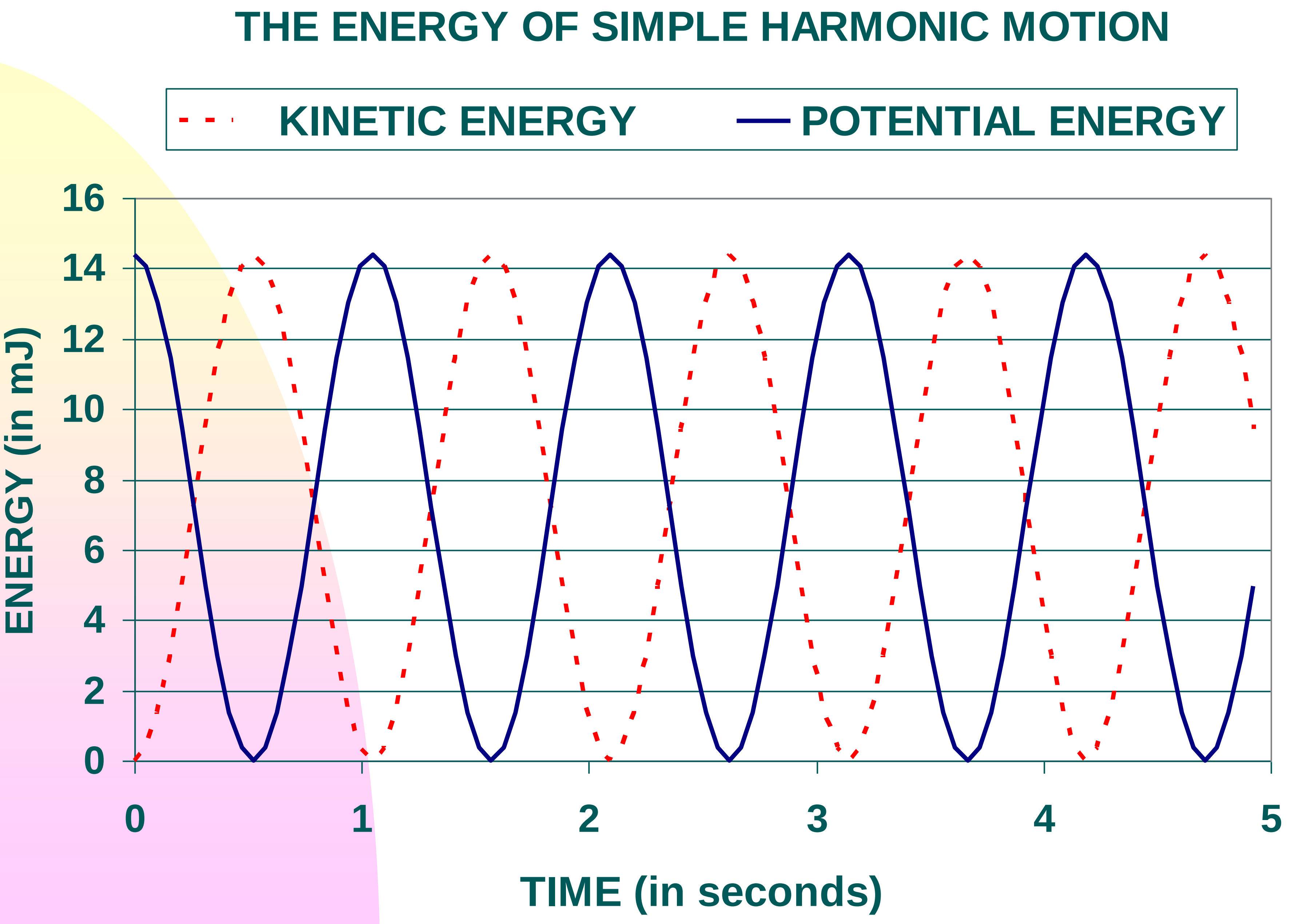
g = acceleration of gravity, 9.80 m/s^2.

h = vertical distance the bob is raised above its lowest position, in meters.

PE = potential energy, in joules.

Conservation of Energy

- Energy for mass on a spring applet
- Energy for pendulum applet


THE ENERGY OF SIMPLE HARMONIC MOTION
KINETIC ENERGY
POTENTIAL ENERGY
ENERGY (in mJ)
16
14
12
10
8
6
4
2
0
0
1
2
3
4
5
TIME (in seconds)

Total Energy: $E = KE + PE$

$$E = \tfrac{1}{2} m v^2 + \tfrac{1}{2} k x^2$$

CONSTANT (assuming there is no friction)

$$E = \tfrac{1}{2} m v_{max}^2$$

because $x = 0$ when $v = v_{max}$

$$E = \tfrac{1}{2} k x_{max}^2$$

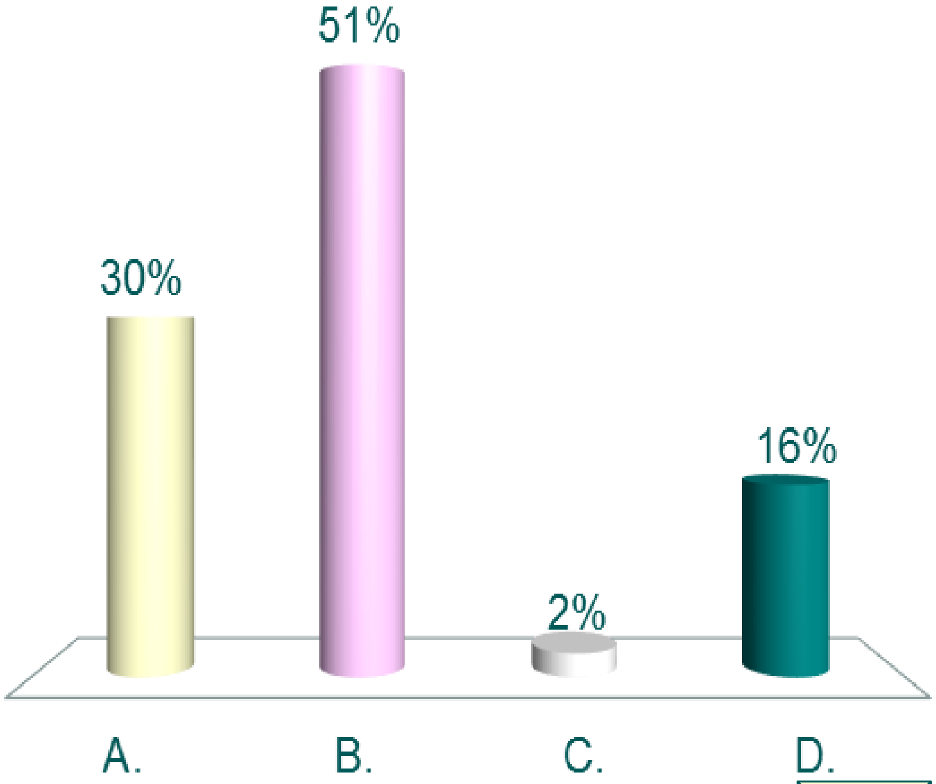
because $v = 0$ when $x = x_{max}$

$$E = \tfrac{1}{2} k A^2$$

where $A = x_{max}$ = amplitude

If the amplitude of an oscillator is tripled, what happens to the energy stored in the oscillation?

A. It is tripled.
B. It increases by a factor of 9.
C. It stays the same.
D. It is cut to a third.



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If the amplitude of an oscillator is tripled, what happens to the energy stored in the oscillation?

A. It is tripled.

B. It increases by a factor of 9.

C. It stays the same.

D. It is cut to a third.

Recall that $E = \frac{1}{2}kA^2$. Because E increases with A^2, E will increase by a factor of nine. Or try a ratio:

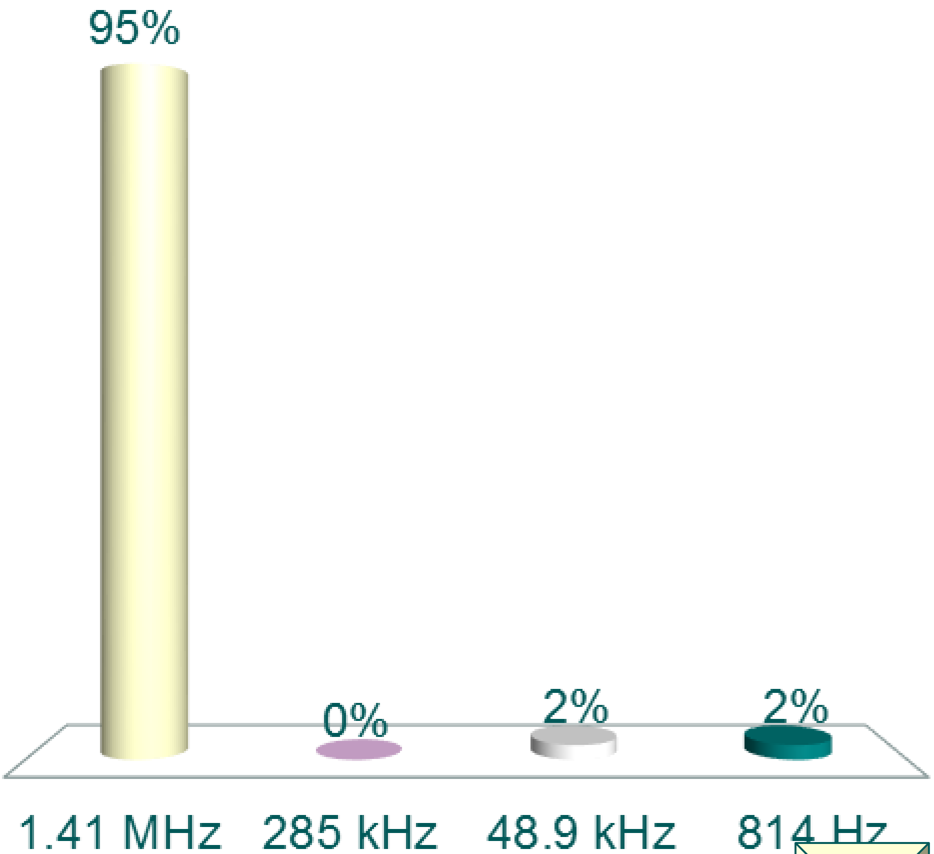
$$\frac{E_{\text{new}}}{E_{\text{old}}} = \frac{\frac{1}{2}k_{\text{new}}A_{\text{new}}^2}{\frac{1}{2}k_{\text{old}}A_{\text{old}}^2} = \frac{\cancel{\frac{1}{2}}\,\cancel{k_{\text{new}}}\left(3\cancel{A_{\text{old}}}\right)^2}{\cancel{\frac{1}{2}}\,\cancel{k_{\text{old}}}\,\cancel{A_{\text{old}}^2}} = \boxed{9}$$

Brief review

- A restoring force produces oscillation. A stronger force (stiffer spring) produces more rapid oscillation.
- The position of an oscillator can be mathematically described by a sine function or a cosine function
- Oscillators store energy, which converts back and forth between kinetic and potential energies

A compact disc encodes music onto 242,000 tiny pits arranged around a circle. If the disc spins at a rate of 350 rev/min, at what frequency are the bits read?

A. 1.41 MHz
B. 285 kHz
C. 48.9 kHz
D. 814 Hz



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A compact disc encodes music onto 285,000 tiny pits arranged around a circle. If the disc spins at a rate of 350 rev/min, at what frequency are the bits read?

A. **1.41 MHz**
B. 285 kHz
C. 48.9 kHz
D. 814 Hz

$$f_{\text{spin}} = 350\,\frac{\text{rev}}{\text{min}} \times \frac{1\ \text{min}}{60\ \text{sec}} = 5.83\,\frac{\text{rev}}{\text{sec}}$$

$$f_{\text{bits}} = Nf_{\text{spin}} = \frac{242{,}000\ \text{bits}}{\text{rev}} \times \frac{5.83\ \text{rev}}{\text{sec}}$$

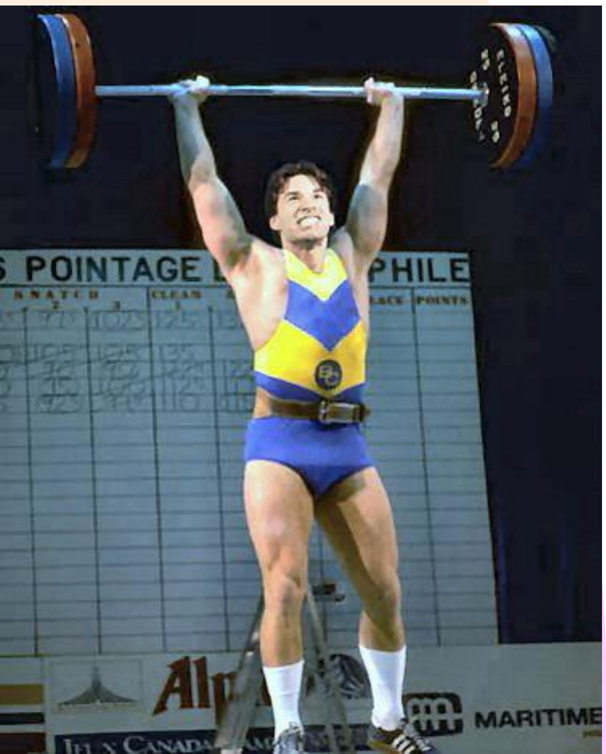
$$= 1.41\times10^{6}\,\frac{\text{bits}}{\text{sec}} = 1.41\times10^{6}\ \text{Hz}$$

$$f_{\text{bits}} = 1.41\times10^{6}\ \text{Hz} \times \frac{1\ \text{MHz}}{1\times10^{6}\ \text{Hz}} = \boxed{1.41\ \text{MHz}}$$

Power

The *rate* at which energy is transferred from one system to another is called the power

$$P = \frac{\text{energy}}{\text{time}} = \frac{\text{joule}}{\text{second}} = \text{watt (W)}$$

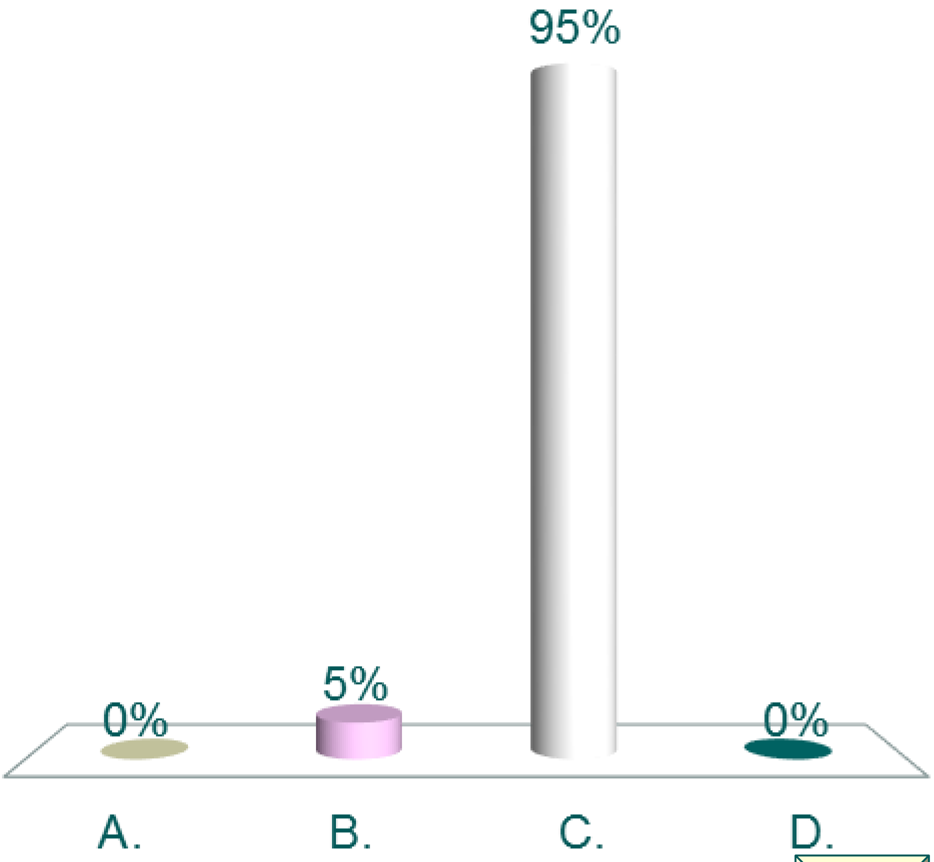


A weightlifter hoists 337 lb (1500 N) a distance of 2.1 m, requiring 3150 J of work. If he does this in 1.80 s, the required power is:

$$P = \frac{3150\ \text{J}}{1.80\ \text{s}} = \boxed{1750\ \text{W}} \times \frac{1\ \text{hp}}{746\ \text{W}} = 2.3\ \text{hp}$$

A loudspeaker generates 4.8 W of acoustic power. How much acoustic energy does it generate in 60 s?

A. 12.5 J
B. 48 J
C. 288 J
D. 1380 J



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A loudspeaker generates 4.8 W of acoustic power. How much acoustic energy does it generate in 60 s?

A. 12.5 J

B. 48 J

C. 288 J

D. 1380 J

$$P = \frac{E}{t}$$

$$Pt = E = \left(4.8\ \frac{\text{J}}{\not{\text{s}}}\right)\left(60\ \not{\text{s}}\right)$$

$$= \boxed{288\ \text{J}}$$